1. Consider the following IVP:
\[
\begin{cases}
  y' = \sqrt{y - 5} \\
  y(a) = b
\end{cases}
\]

Use the theorem of existence to determine for what values of \( b \) there are: infinitely many solutions, a unique solution and no solution.

2. Consider the following IVP:
\[
\begin{cases}
  y' = \ln(y - 3) \\
  y(a) = b
\end{cases}
\]

Use the theorem of existence to determine for what values of \( b \) there are: infinitely many solutions, a unique solution and no solution.
3. Solve the following differential equation:

\[ x^2 y' = 1 - x^2 + y^2 - x^2 y^2 \]

4. Solve the following differential equation:

\[ \frac{dy}{dx} = 3\sqrt{xy} \]
5. Solve the following differential equation:

\[ 2xyy' = x^2 + 2y^2 \]

6. Solve the following differential equation:

\[ \frac{xy'}{y} = 4x^2 + \ln y \]
7. Consider the following two functions: \( y_1 = x^2 \) and \( y_2 = x^{-3} \). Determine where \( y_1 \) and \( y_2 \) are linear independent and show that they both solve the following differential equation:

\[
x^2 y'' + 2xy' - 6y = 0
\]

8. Consider the following two functions: \( y_1 = x \) and \( y_2 = x \ln x \). Determine where \( y_1 \) and \( y_2 \) are linear independent and show that they both solve the following differential equation:

\[
x^2 y'' - xy' + y = 0
\]
9. (BONUS) Let $f(x)$ be a real valued differentiable function on all of $\mathbb{R}$. Define $u(x)$ in the following way

$$u(x) = \sin x \int_0^x f(t) \cos t \, dt - \cos x \int_0^x f(t) \sin t \, dt$$

Verify that $u$ is a solution to the following differential equation:

$$\frac{d^2 u}{dx^2} + u = f(x)$$