1. For each of the numbers $a, b, c, d, r,$ and $s$, state whether the function whose graph is shown has an absolute maximum or minimum, a local maximum or minimum, or neither a maximum nor a minimum.

\[ y \]
\[ 0 \quad a \quad b \quad c \quad d \quad r \quad s \quad x \]

Solution. Let the function be $f(x)$.

The function has its absolute minimum at $x = a$. Indeed, $f(a)$ is the smallest of all the $y$-values on the graph. However, since $x = a$ is an endpoint of the domain of the function, we do not consider it a local minimum. This may seem a bit strange, but endpoints are not local maxima or minima, so that one can have an absolute maximum or minimum, but not a local one.

At $x = b$, $f$ has a local maximum. Indeed, $f(b)$ is larger than $f(x)$ for any $x$ slightly below or slightly above $b$. You should check this yourself.

For the same reason as above, $f$ has a local maximum at $x = r$. Furthermore, $f(r)$ is the largest $y$-value in the domain of $f$, so that $f$ has its absolute maximum at $x = r$.

At $x = c$, $f$ has neither a local minimum nor a local maximum. Indeed, if $x$ is slightly smaller than $c$, we have that $f(x) > f(c)$, so that $x = c$ cannot be a local maximum. On the other hand, if $x$ is slightly larger than $c$, we have that $f(x) < f(c)$, so that $x$ cannot be a local minimum. Therefore, it is neither.

At $x = d$, $f$ has a local minimum, as $f(d)$ is smaller than $f(x)$ for any $x$ near $d$. Again, you should check this yourself.
Finally, at $x = s$, we have neither a local maximum or minimum (since $x = s$ is an endpoint in the domain of $f$). $\square$

2. Use the graph to state the absolute and local maximum and minimum values of the function.

![Graph of $f(x)$]

Solution. Again, let the function in the graph be $f$.

Note first that $f$ has a local minimum at $x = 1$. Indeed, $f(x)$ is smaller than $f(1)$ for all $x$-values close to 1, so $x = 1$ fits the definition.

It is probably also readily apparent to you that $f$ has a local minimum at $x = 2$ and $x = 5$, and a local maximum at $x = 4$.

Next, note that $f$ has a local maximum at $x = 6$. As in the first problem, if we choose an $x$-value either slightly less than or slightly greater than 6 (check each individually!), we see that $f(x) \leq f(6)$, so that $f(6)$ is a local maximum for $f$. This catches all the local maxima or minima of $f$.

It is clear that $f$ has its absolute maximum at $x = 4$, as $f(4)$ is the largest $y$-value in the domain of $f$. However, $f$ does not have an absolute minimum, as we discussed in class. $\square$

3. Sketch the graph of a function $f$ that is continuous on $[1, 5]$, has an absolute minimum at 1, an absolute maximum at 5, a local maximum at 2, and a local minimum at 4.

4. Sketch the graph of a function on $[-1, 2]$ that has an absolute maximum but no absolute minimum.
5. Sketch the graph of a function on \([-1, 2]\) that is discontinuous but has both an absolute maximum and an absolute minimum.

Solution. There are some nice examples sketched out in the back of your book that show the types of graphs desired in 3, 4, and 5. See me if you need additional information. □

6. Find the critical numbers of \(f(p) = \frac{p-1}{p^2-4}\).

Solution. Recall that the critical numbers of \(f\) are the \(x\)-values in the domain of \(f\) at which \(f' = 0\) or \(f'\) does not exist. We have:

\[
f'(p) = \frac{(p^2 - 4) - 2p(p - 1)}{(p^2 - 4)^2} = \frac{-p^2 - 2p - 4}{(p^2 - 4)^2}
\]

Note that \(f'(p)\) does not exist when the denominator is 0, i.e. when \(x = 2\) or \(x = -2\). Of course, neither of these is in the domain of \(f\), so that neither is a critical number of \(f\).

On the other hand, \(f'(p) = 0\) precisely when \(-p^2 - 2p - 4 = 0\), Using the quadratic formula, we have that this happens for:

\[
p = \frac{2 \pm \sqrt{4 - 16}}{-2}
\]

Of course, neither of these values is real, so that \(f\) has no critical numbers. □

7. Find the absolute maximum and minimum values of \(f(x) = 12 + 4x - x^2\) on the interval \([0, 5]\).

Solution. Recall our method: we evaluate \(f\) at its critical numbers, at 0, and at 5. We begin by finding the critical numbers of \(f\). We have:

\[
f'(x) = 4 - 2x
\]

Note that \(f'(x)\) always exists on \([0, 5]\), and that \(f'(x) = 0\) for \(x = 2\). Therefore, we have \(f(0) = 12\), \(f(2) = 16\), and \(f(5) = 7\), so that the absolute maximum value of \(f\) on \([0, 5]\) is \(f(2) = 16\), and the absolute minimum value of \(f\) on \([0, 5]\) is \(f(5) = 7\). We have the result. □