1. \[ \sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 1} \]

Let \( b_n = \frac{n}{n^2 + 1} \). Then \( b_n \geq 0 \) for all \( n = 1, 2, 3, \ldots \).

\[ \text{and } b_n \text{ is decreasing} \]

\[ \lim_{n \to \infty} b_n = \lim_{n \to \infty} \frac{n}{n^2 + 1} = \lim_{n \to \infty} \frac{1}{n + \frac{1}{n}} = 0 < 1. \]

So \( \sum (-1)^n n \) converges by the Alternating Series Test.

But \( \sum \frac{n}{n^2 + 1} \) diverges by the Limit Comparison Test: since \( \sum \frac{1}{n} \)

diverges and \( \lim_{n \to \infty} \frac{\frac{n}{n^2 + 1}}{\frac{1}{n}} = \lim_{n \to \infty} \frac{n^2}{n^2 + 1} = \lim_{n \to \infty} \frac{1 + \frac{1}{n^2}}{1 + \frac{1}{n^2}} = 1 \neq 0. \)

So the series is conditionally convergent.

2. \[ \sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2 + 1} \]

Since \( \lim_{n \to \infty} \frac{n^2}{n^2 + 1} = 1 \), then \( \frac{(-1)^n n^2}{n^2 + 1} \) does not have a limit of 0 (it bounces back and forth between numbers getting closer to 1 and numbers getting closer to (-1).)

So the series diverges, by the Theorem which says that a series \( \sum a_n \) can only converge if \( \lim a_n = 0 \).

3. \[ \sum_{n=1}^{\infty} \frac{2^n \cos n}{n!} \]

Here \( a_n = \frac{2^n \cos n}{n!} \), so \( |a_n| \leq \frac{2^n}{n!} \).

But \( \sum \frac{2^n}{n!} \) converges by the Ratio Test, since

\[ \lim_{n \to \infty} \frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} = \lim_{n \to \infty} \frac{2}{n+1} = 0 < 1. \]

So \( \sum |a_n| \) converges by the Comparison Test.

Therefore \( \sum a_n \) converges absolutely.