The shaded region in the diagram lies to the right of the y-axis, between the line $y = 2x$ and the curve $y = x^3 - 7x$. Find the volume of the solid obtained by rotating the region around the line $x = -1$.

The element of integration is a shell with height $h = PQ$ and radius $r = AP$ and thickness $\Delta x$. The volume of the shell is approximately $2\pi rh \Delta x$.

The rightmost shell is at $S$, whose coordinates are found by setting $2x = y = x^3 - 7x$. This gives $x^3 - 9x = 0$ so $x^3 = 9x$, so $x^2 = 9$, so $x = \pm 3$. Here $x = 3$.

The leftmost shell is at $x = 0$.

So the volume of the solid is

$$V = \int_{x=0}^{x=3} 2\pi rh \, dx =$$

$$= \int_0^3 2\pi \left(2x - (x^3 - 7x)\right) (x+1) \, dx$$

(since $r = AP = x + 1$

$h = PQ = 2x - (x^3 - 7x)$)

$$= \int_0^3 2\pi \left(9x - x^3\right) (x+1) \, dx$$

$$= 2\pi \int_0^3 \left(9x^2 - x^4 + 9x - x^3\right) \, dx =$$

$$= 2\pi \left[3x^3 - \frac{x^5}{5} + \frac{9x^2}{2} - \frac{x^4}{4}\right]_0^3 = 2\pi \left[3 \cdot 3^3 - \frac{3^5}{5} + \frac{9 \cdot 3^2}{2} - \frac{3^4}{4}\right]$$

$$= 2\pi \left[\frac{3^4}{4} - \frac{3^5}{5} + \frac{3^3}{2} - \frac{3^4}{4}\right] = \frac{653\pi}{10}$$