Quiz 5

Name: key

1. Use a linear approximation (or differentials) to estimate \( \sqrt{49.01} \). Show your work.

Let \( x = 49.01 \), \( f(x) = \sqrt{x} \), and \( \bar{x} = 49 \).

Then \( f(x) \approx f(\bar{x}) + (x - \bar{x}) f'(\bar{x}) \).

So \( \sqrt{49.01} \approx \sqrt{49} + (0.01) \frac{1}{2\sqrt{49}} \).

Since \( f'(x) = \frac{1}{2\sqrt{x}} \), then \( f'(49) = \frac{1}{14} \).

So \( \sqrt{49.01} \approx 7 + (0.01) \frac{1}{14} = 7 + \frac{1}{1400} \).

(Here's how I explained it in class:

\[ f(x) = \sqrt{x} \]

\[ f'(x) = \frac{1}{2\sqrt{x}} \]

\[ P = (49, 7) \]

\[ Q = (49.01, 7) \]

\[ QS = \frac{1}{14} (0.01) \]

\[ \overline{QR} = \text{slope of PR}, \ P Q \]

\[ QS \approx f'(7), \ PQ \]

\[ QS \approx \frac{1}{14} \times (0.01) \]

\[ \sqrt{49.01} \approx 7 + QS = 7 + \frac{0.01}{14} \]

2. If \( f''(x) = \sin x \), \( f'(0) = 7 \), and \( f(0) = 3 \), find \( f(x) \).

Since \( f''(x) = \sin x \),

Then \( f'(x) = -\cos x + C \) for some constant \( C \).

Since \( f'(0) = 7 \), then

\[ 7 = -\cos 0 + C \], so \( 7 = -1 + C \), so \( C = 8 \).

Hence \( f'(x) = -\cos x + 8 \).

Therefore \( f(x) = -\sin x + 8x + D \).

Since \( f(0) = 3 \), then

\[ 3 = -\sin 0 + 8 \cdot 0 + D \], so \( 3 = D \).

So \( f(x) = -\sin x + 8x + 3 \).