1. Find the area of the bounded region contained between the graphs of $y^2 - x = 4$ and $y = x$.

2. Prove that the volume of a sphere of radius $r$ is $\frac{4}{3}\pi r^3$. 
3. A solid is contained between the planes \( x = 0 \) and \( x = 10 \). When the solid is sliced by the plane perpendicular to the \( x \)-axis with \( x \)-coordinate \( x \), the resulting cross-section is an equilateral triangle with sides of length \( x \). Find the volume of the solid.

4. Find the volume of the solid obtained by rotating the region bounded by the curves \( y = x^2 \), \( y = x \) about the line \( x = 3 \).
5. A tank shaped like an upside down cone is filled with water. The height of the cone is $h$ and the radius of the base is $r$. The density of water is 1000 kg per cubic meter. Water is pumped out over the top. How much work is required to empty the tank?

6. Let $p(t)$ denote the position of a particle as a function of time $t$, for $a \leq t \leq b$. Show that the average velocity of the particle on the interval $[a, b]$ is equal to the average value of the velocity function $v(t) = p'(t)$ on the interval.
7. Let \( f : [a, b] \rightarrow [c, d] \) be a one-to-one and onto function. Let \( g : [c, d] \rightarrow [a, b] \) denote the inverse. Assume \( 0 \leq a < b \) and \( 0 \leq c < d \). The area under the graph of \( f \) is \( A \). Find the area under the graph of \( g \).

8. Let \( f : \mathbb{R} \rightarrow \mathbb{R} \) be a function that has an inverse \( f^{-1} \). Does \( f^2 \) have an inverse? Why or why not? If so, what is the inverse? Answer the same questions for \( f^3 \).
9. Evaluate the following integrals

a) \[ \int \tan x \, dx \]

b) \[ \int \frac{\sin x}{e^{\cos x}} \, dx \]

c) \[ \int (e^u + e^{-u})^2 \, du \]

d) \[ \int \frac{x}{-x^2 + 1} \, dx \]

e) \[ \int \frac{t^2}{t + 2} \, dt \]

f) \[ \int 2^x \cdot 3^x \, dt \]
10. Find the derivatives of the following functions

a) \(x^2 2^x\)

b) \(e^x e^x\)

c) \(x \cos x\)

d) \(\frac{x^{3/4} (x - 1)^2}{(\cos x)^3}\)

e) \(x \ln |x| - x\)

f) \(\ln \left(x^2 (\cos x) \sqrt{x + 1}\right)\)

g) \(\log_2 (5x^2 + 1)\)

h) \(\ln |2x + \tan x|\)

i) \((\ln x)^3\)
11. Recall that \( \ln \) is the function defined by

\[
\ln x = \int_{1}^{x} \frac{1}{t} \, dt.
\]

a) Show that \( \ln \) is an increasing function.

b) Show that \( \ln \) is concave down.

c) Explain why \( \ln \) is a continuous function.