Discrete Mathematics
Review Counting Problems with answers

1. How many ways are there to select a group of 5 women from 16 husband/wife teams?
   answer: 4368

2. How many ways are there to arrange the 7 letters AAABBBB?
   answer: 35

3. How many ways are there to arrange the 12 letters of AAABBBBCCCCC without having two C’s together?
   answer: 1960

4. How many ways are there to seat 10 people in a row?
   answer: 3,628,800

5. How many ways are there to seat 10 people at a round table?
   answer: 362,880

6. How many distinguishable dominoes are there? (Each of the two ends of domino has 0 to 6 dots on it.)
   answer: 28

7. How many ways can 14 men and 9 women be seated in a row so that no 2 women sit next to each other?
   answer: 436,327,347,456,000

8. How many ways are there to select 10 cans of soda from 4 different brands?
   answer: 286

9. How many ways can 22 cans of beer be handed out to 4 people if everyone must get at least one can?
   answer: 1,330

10. How many ways are there to pick 9 cans from a tub containing 8 cans of 57 different varieties of soda?
    answer: 31,966,749,823

11. How many ways are there to distribute 5 apples and 8 oranges to 6 children?
    answer: 324,324

12. How many ways are there to select some fruit from 5 apples and 8 oranges, taking at least one piece?
    answer: 53

13. How many nonnegative integers less than a billion have five 7’s?
    answer: 826,686

14. How many 5-letter words can be formed from the alphabet without repeating any letter?
    answer: 7,893,600

15. How many ways are there to pair off 8 women and 8 men at a dance?
    answer: 40,320

16. How many positive integer solutions are there to the equation \( w + x + y + z = 24 \)?
    answer: 1771

17. How many ways are there to pick 12 letters from 12 A’s and 12 B’s?
    answer: 13

18. How many ways are there to pick 18 letters from 12 A’s and 12 B’s?
    answer: 7

19. How many ways are there to pick 25 letters from 12 A’s, 12 B’s and 12 C’s?
    answer: 78
20. How many ways are there to select a dozen doughnuts chosen from 7 varieties with the restriction that at least 1 doughnut of each variety is chosen?
   answer: 462

21. How many ways are there to assign 50 agents to 5 different countries so that each country gets 10 agents?
   answer: 48334775757901219912115629238400

22. How many ways are there to put 17 red balls into 12 distinguishable boxes with at least 1 ball in each box?
   answer: 4368

23. How many ways can 9 dice fall (unordered)?
   answer: 2002

24. How many ways are there to arrange 5 C’s and 15 R’s such that there are at least 2 R’s between any 2 C’s?
   answer: 792

25. How many ways are there to select 5 integers from \( \{1, 2, \ldots, 20\} \) such that the (positive) difference between any two of the five is at least 3?
   answer: 792

26. How many possible outcomes (unordered) are there if \( k \) dice are tossed?
   answer: \( C(k + 5, k) \)

27. How many bit strings of length 5 are there that either start with 000 or end with 111?
   answer: 8

28. How many bit strings of length \( n \) where \( n > 5 \) are there that either start with 000 or end with 111?
   answer: \( 15 \cdot 2^{n-6} \)

29. How many 4-letter words are there with the letters in alphabetical order?
   answer: 23,751

30. How many 4-letter words are there with no letter repeated and the letters in alphabetical order?
   answer: 14,950

31. How many ways can we partition 18 persons into study groups of 5, 6 and 7?
   answer: 14,702,688

32. How many ways can we partition 18 persons into study groups of 5, 5 and 4?
   answer: 0

33. How many ways can we partition 18 persons into 3 study groups of 6?
   answer: 2,858,856

34. How many arrangements of 7 R’s and 11 B’s are there such that no two R’s are adjacent?
   answer: 792

35. How many ways are there to give each of 5 children 4 of 20 distinguishable toys?
   answer: 305,540,235,000

36. How many ways can 10 men and 7 women sit in a row so that no two women are next to each other?
   answer: 6,035,420,160,000

37. How many ways can 10 men and 7 women sit at a round table so that no two women are next to each other?
   answer: 219,469,824,000

38. How many 3-letter words are there with no repeated letter if the middle letter is a vowel?
   answer: 3,000
39. How many 5-card poker hands are there with two pairs?
   \textbf{answer:} 123,552

40. How many arrangements of the letters in MISSISSIPPI have at least two adjacent I’s?
   \textbf{answer:} 27,300

41. How many arrangements of the letters in MISSISSIPPI have no two I’s adjacent?
   \textbf{answer:} 7,350

42. How many arrangements of the letters in MISSISSIPPI have no P adjacent to an S? (Hint: Although it is the same problem, it is easier to consider no S adjacent to a P.)
   \textbf{answer:} 4725

43. How many possible outcomes are there if a pair of dodecahedral dice, with sides numbered 1 through 12, are thrown?
   \textbf{answer:} 78

44. How many different selections of fruit can be made from 5 oranges and 7 apples?
   \textbf{answer:} $6 \cdot 8 = 48$

45. How many different words of at least one letter can be made from 3 A’s and 3 B’s?
   \textbf{answer:} 68

46. How many ways can we partition $mn$ distinguishable objects into $m$ piles of $n$ objects each?
   \textbf{answer:} $\left(\begin{array}{c} mn \\ n, n, \ldots, n \end{array}\right)/m!

47. How many 5-letter words using only A’s, B’s, C’s and D’s are there that do not contain the word BAD?
   \textbf{answer:} 976

48. How many 10-letter words using only A’s, B’s, C’s and D’s are there that either start or end with BAD are there?
   \textbf{answer:} 32,512

49. How many 10-letter words using only A’s, B’s, C’s and D’s are there which have 3 A’s, 2 B’s and 3 C’s but do not contain the word AB?
   \textbf{answer:} 11,760

50. How many bit strings of length twelve don’t include a ‘01’ substring?
   \textbf{answer:} 13

51. How many bit strings of length 12 don’t contain a ‘11’ substring?
   \textbf{answer:} 377

52. How many nonnegative integer solutions are there to the equation
   \[ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 32? \]
   \textbf{answer:} 435,897

53. How many positive integer solutions are there to the equation
   \[ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 32? \]
   \textbf{answer:} 169,911

54. How many positive integer solutions are there to the inequality
   \[ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 < 32? \]
   (Hint: consider $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 32$.)
   \textbf{answer:} 736,281
55. How many nonnegative integer solutions are there to the inequality
\[ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 < 32? \]
answer: 2,324,784

56. How many arrangements of the letters of RECURRENCERELATION have no two vowels adjacent?
answer: 1,309,770,000

57. How many arrangements of the letters of RECURRENCERELATION have the vowels in alphabetical order?
answer: 1,202,947,200

58. How many ways can 8 persons, including Peter and Paul, sit in a row with Peter and Paul not sitting next to each other?
answer: 30,240

59. How many ways can 8 persons, including Peter and Paul, sit at a round table with Peter and Paul sitting next to each other?
answer: 1,440

60. How many ways can 4 persons of each of \( n \) nationalities stand in a row with each person standing next to a fellow national?
answer: \((2n)!(4!)^n/2^n \)

61. How many ways are there to distribute 30 green balls to 4 persons if Alice and Eve together get no more than 20 and Lucky gets at least 7?
answer: 2464