
Homework 9

Due: Friday, 22 April, 2016.

Practice: Questions marked with a \checkmark in Sections Four.I.1–2 and Four.II.1 from the textbook.

1. [20 marks] Calculate the following determinants.

$$\begin{vmatrix} 3 & 6 \\ -1 & 3 \end{vmatrix}, \quad \begin{vmatrix} 2 & 0 & 3 \\ 0 & 1 & 2 \\ 1 & 1 & 2 \end{vmatrix}, \quad \begin{vmatrix} 1 & 3 & 2 \\ 3 & 1 & 5 \\ 2 & 6 & 5 \end{vmatrix}, \quad \begin{vmatrix} 0 & 0 & 2 & 3 \\ 0 & 1 & 6 & 4 \\ 0 & 0 & 0 & 2 \\ 3 & 1 & 5 & 3 \end{vmatrix}.$$

2. [20 marks] Let $C = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid x^2 + y^2 = 1 \right\}$ be the unit circle in the plane. Let

$T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be linear transformation given by $T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax \\ by \end{pmatrix}$, where $a, b > 0$.

- (a) Write down a formula for T^{-1} .
- (b) Show that the image of C under T is the ellipse E given by the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

(Hint: observe that a point $\begin{pmatrix} x \\ y \end{pmatrix}$ lies on E if and only if $T^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$ lies on C .)

- (c) Calculate $\det T$, and hence find the area enclosed by E . (Hint: first find the area enclosed by C , then scale accordingly.)