Homework 7

Due: Friday, 1 April, 2016.
Practice: Questions marked with a $\surd$ in Sections Three.IV.4 from the textbook.

1. [**15 marks**] Determine whether the following matrices are invertible and, if so, find the inverse.

$$A = \begin{pmatrix} 6 & 5 \\ 5 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 5 & 0 \\ 3 & 3 & 8 \end{pmatrix}, C = \begin{pmatrix} 2 & 3 & 6 \\ 4 & 1 & 9 \\ 0 & 5 & 3 \end{pmatrix}, D = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

2. [**6 marks**] Use part (a) to solve the the following SLE's. (Just do an appropriate matrix multiplication – no need to solve these from scratch.)

(a)
$$\begin{aligned} 6x + 5y &= 5 \\ 5x + 4y &= 7 \end{aligned}$$

(b)
$$\begin{aligned} x + 2y + z &= 2 \\ 2x + 5y &= -1 \\ 3x + 3y + 8z &= 4 \end{aligned}$$

(c)
$$\begin{aligned} x + 2y + z &= 3 \\ 2x + 5y &= 5 \\ 3x + 3y + 8z &= -2 \end{aligned}$$

3. [**5 marks**] Determine whether the derivative operator $\frac{d}{dx} : \mathcal{P} \to \mathcal{P}$ is invertible, where $\mathcal{P}$ is the vector space of real polynomials.

4. (a) [**4 marks**] An $m \times n$ matrix A is called *symmetric* if $A^T = A$. For example, any diagonal matrix is symmetric. Give an example of a symmetric 3×3 matrix which is not a diagonal matrix. What relationship between m and n must hold if A is symmetric?

(b) [**5 marks**] Let A be a $m \times n$ matrix. Prove that $A^T A$ is symmetric. (Hint: take the transpose of $A^T A$.)

(c) [**5 marks**] Suppose A is an invertible matrix. Prove that $(A^{-1})^T = (A^T)^{-1}$. (Hint: take the transpose of $A^{-1}A$.)