Homework 3

Due: Friday, 12 February, 2016.
Practice: Questions marked with a $\surd$ in Sections Two.I.1–2 from the textbook.

1. [**8 marks**] Classify all 2×3 matrices in reduced row echelon form. They should fall into 7 "families" according to the positions of their leading 1's. For example, $\begin{pmatrix} 1 & a & 0 \\ 0 & 0 & 1 \end{pmatrix}$ where $a \in \mathbb{R}$, describes one such family. Is it possible for a SLE with 2 equations in 3 variables to have a unique solution?

2. [**10 marks**] Show that the following are *not* vector spaces. (i.e. find a vector space axiom which is violated.)

(a) The union of the x–axis and y–axis in $\mathbb{R}^2$ with the inherited operations.

(b) $\{p \in \mathcal{P}_2 \mid p(2) = 4\}$ with the operations inherited from $\mathcal{P}_2$, the set of polynomials of degree at most 2.

3. [**8 marks**] Let V be a vector space, and $\mathbf{v} \in V$ be a vector. Prove, using the vector space axioms, that the negative of $\mathbf{v}$ is unique. That is, show that if $\mathbf{u}, \mathbf{u}' \in V$ satisfy $\mathbf{v} + \mathbf{u} = \mathbf{0}$ and $\mathbf{v} + \mathbf{u}' = \mathbf{0}$ then $\mathbf{u} = \mathbf{u}'$. Indicate which axiom you are using at each step. (Note: you cannot assume V is a specific vector space such as $\mathbb{R}^n$.)

4. [**14 marks**] Let V be the set of real numbers equipped with vector addition $\oplus$ and scalar multiplication $\odot$ defined as follows: for all $x, y \in V$ and $r \in \mathbb{R}$, let

- $x \oplus y = x + y - 2$
- $r \odot x = r(x - 2) + 2$.

(a) Compute $0 \oplus 0$, $1 \oplus 2$, $0 \odot x$, $(-1) \odot x$.

(b) Prove that V equipped with $\oplus$ and $\odot$ is a vector space. What is the zero vector in V? What is the negative of x in V?