Homework 2

Due: Friday, 5 February, 2016.
Practice: Questions marked with a $\surd$ in Sections One.III.1–2 from the textbook.

1. [**12 marks**] Let $A = \begin{pmatrix} 1 & 2 & 5 \\ 3 & 4 & 5 \\ 1 & 0 & -5 \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, and $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$.

(a) Write the solution set of $A\mathbf{x} = \mathbf{b}$ in vector form, and identify a particular solution.

(b) Use your answer to part (a) to write down the solution set of $A\mathbf{x} = \mathbf{0}$.

2. [**20 marks**] Convert the following matrices to reduced row echelon form. Find all pairs of row equivalent matrices. Which ones are singular/non-singular/neither?

$$A = \begin{pmatrix} 1 & 2 & 5 \\ 3 & 6 & 1 \\ 2 & 4 & 3 \end{pmatrix},\ B = \begin{pmatrix} 4 & -2 & 5 \\ -2 & 1 & 2 \\ 3 & 0 & 3 \end{pmatrix},\ C = \begin{pmatrix} 4 & -2 & 5 \\ -2 & 1 & 2 \end{pmatrix},\ D = \begin{pmatrix} 4 & 8 & 0 \\ 2 & 4 & -3 \\ -1 & -2 & 7 \end{pmatrix}.$$

3. [**8 marks**] Let $\mathbf{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$ be $n \times 1$ vectors, and A be a $m \times n$ matrix. Recall that

$$A\mathbf{x} = x_1\boldsymbol{\alpha}_1 + x_2\boldsymbol{\alpha}_2 + \ldots + x_n\boldsymbol{\alpha}_n$$

where $\boldsymbol{\alpha}_j$ is the jth column of A. Show that $A(\mathbf{x}+\mathbf{y}) = A\mathbf{x} + A\mathbf{y}$ and $A(c\mathbf{x}) = c(A\mathbf{x})$ for all $c \in \mathbb{R}$.