Homework 9

Due: Friday, 29 April, 2016.
Practice: Questions marked with a $\surd$ in Sections Four.II.2–3 from the textbook.

1. [**24 marks**] For the following matrices, compute the characteristic polynomial and hence find the eigenvalues. For each eigenvalue, find as many linearly independent eigenvectors as possible. If the matrix is diagonalisable, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. Write down the matrices A_1^{100} and A_2^{100} (you may leave powers of numbers unsimplified).

$$A_1 = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}, \qquad A_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad A_4 = \begin{pmatrix} 3 & 1 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

2. [**16 marks**]

(a) Suppose A and B are similar $n \times n$ matrices, i.e. there exists an invertible matrix P such that $A = PBP^{-1}$. Show that A and B have the same characteristic polynomials, and therefore the same eigenvalues. (Hint: show that for any scalar λ, the matrices $A - \lambda I$ and $B - \lambda I$ are also similar. Then take determinants.)

(b) Let A and B be 2×2 matrices with the same eigenvalues. Must A and B be similar? If yes, give reasons; if not, give an example.