Homework 1

Due: Friday, 29 January, 2016.
Practice: Questions marked with a $\surd$ in Sections One.I.1–3 from the textbook.

1. [**16 marks**] Solve the following systems of linear equations (SLE's) by first writing down the associated augmented matrix, then applying Gauss' Method. Express the solution sets in vector form. You should also indicate which elementary row operations you use.

   (a)
   $$\begin{aligned} 2x + y - z &= 5 \\ 4x + 3y - z &= 1 \\ x + y &= 2 \end{aligned}$$

   (b)
   $$\begin{aligned} x + 2y - z + 3w &= 2 \\ x + y - z + w &= -1 \\ 3x + 5y + 3z + 7w &= 3 \end{aligned}$$

2. [**8 marks**] Can $\begin{pmatrix} 4 \\ 8 \\ 4 \end{pmatrix}$ be expressed as a linear combination of $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, and $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$? If so, is there a unique such linear combination?

3. [**8 marks**] For what values of λ does the following SLE have more than one solution?
   $$\begin{aligned} (2-\lambda)\,x + 6y &= 0 \\ x + (1-\lambda)\,y &= 0 \end{aligned}$$

4. [**8 marks**] Let $\mathbf{u} = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$ be vectors in $\mathbb{R}^n$ which both satisfy the following SLE:
   $$\begin{aligned} a_{1,1}x_1 + a_{1,2}x_2 + \cdots + a_{1,n}x_n &= b_1 \\ a_{2,1}x_1 + a_{2,2}x_2 + \cdots + a_{2,n}x_n &= b_2 \\ &\vdots \\ a_{m,1}x_1 + a_{m,2}x_2 + \cdots + a_{m,n}x_n &= b_m \end{aligned}$$
   Show that any vector of the form $\mathbf{r}(t) = t\mathbf{u} + (1-t)\mathbf{v}$, for $t \in \mathbb{R}$, also satisfies the above SLE. (This proves that the straight line through two distinct points in the solution set of a SLE must also lie in that solution set.)