List of Formulae

The following are provided without the conditions under which they hold.

Discriminant

If $f(x,y)$ is a function on $\mathbb{R}^2$, then $D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - (f_{xy}(a,b))^2$.

Change of co-ordinates

- Polar co-ordinates: $x = r\cos\theta,\ y = r\sin\theta,\ dA = r\ dr\ d\theta.$
- Cylindrical co-ordinates: $x = r\cos\theta,\ y = r\sin\theta,\ z = z,\ dV = r\ dz\ dr\ d\theta.$
- Spherical co-ordinates: $x = \rho\sin\phi\cos\theta,\ y = \rho\sin\phi\sin\theta,\ z = \rho\cos\phi,\ dV = \rho^2\sin\phi\ d\rho\ d\phi\ d\theta.$

div, grad and curl

Let $f(x,y,z)$ be a scalar function and $\mathbf{F} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$ be a vector field on $\mathbb{R}^3$.

- $\operatorname{grad} f = \nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}.$
- $\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}.$
- $\operatorname{curl}\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = (\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z})\mathbf{i} + (\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x})\mathbf{j} + (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})\mathbf{k}.$

Line and surface integrals

Let C be a curve in $\mathbb{R}^2$ or $\mathbb{R}^3$ parameterised by $\mathbf{r}(t)$, $a \le t \le b$.
Let S be a surface in $\mathbb{R}^3$ with parameterisation $\mathbf{r}(u,v) = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k}$, $(u,v) \in D$.

- $\int_C f\ ds = \int_a^b f(\mathbf{r}(t))\ |\mathbf{r}'(t)|\ dt,$
- $\int_C \mathbf{F}\cdot d\mathbf{r} = \int_C (\mathbf{F}\cdot\mathbf{T})\ ds = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\ dt,$
- $\iint_S f\ dS = \iint_D f(\mathbf{r}(u,v))\ |\mathbf{r}_u \times \mathbf{r}_v|\ dA,$
- $\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_S (\mathbf{F}\cdot\mathbf{n})\ dS = \iint_D \mathbf{F}(\mathbf{r}(u,v))\cdot(\mathbf{r}_u \times \mathbf{r}_v)\ dA,$

where $\mathbf{r}_u = \frac{\partial \mathbf{r}}{\partial u} = \frac{\partial x}{\partial u}\mathbf{i} + \frac{\partial y}{\partial u}\mathbf{j} + \frac{\partial z}{\partial u}\mathbf{k}$ and $\mathbf{r}_v = \frac{\partial \mathbf{r}}{\partial v} = \frac{\partial x}{\partial v}\mathbf{i} + \frac{\partial y}{\partial v}\mathbf{j} + \frac{\partial z}{\partial v}\mathbf{k}$.

Vector Calculus Theorems

Let C be a curve, D a plane region, S a surface, and E a solid region. The symbol ∂ indicates the positively oriented boundary.

- Fundamental Theorem for line integrals: $\int_C \nabla f\cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a)).$
- Green's Theorem: $\iint_D (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})\ dA = \int_{\partial D} P\ dx + Q\ dy.$
- Stokes' Theorem: $\iint_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} = \int_{\partial S}\mathbf{F}\cdot d\mathbf{r}.$
- Divergence Theorem: $\iiint_E \operatorname{div}\mathbf{F}\ dV = \iint_{\partial E}\mathbf{F}\cdot d\mathbf{S}.$