1. Perform vector addition, subtraction, and scalar multiplication using (a) a diagram, and (b) in terms of components. When are two vectors equal? When are they parallel? How do you calculate the magnitude of a vector? What is a unit vector? How do you find a unit vector parallel to a given vector? What is a position vector? What is a displacement vector? How do you calculate the displacement vector $\overrightarrow{AB}$ in terms of position vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$?

2. Define the dot product $\mathbf{u} \cdot \mathbf{v}$ of two vectors $\mathbf{u}$ and $\mathbf{v}$. Is it a scalar or vector? How do you compute $\mathbf{u} \cdot \mathbf{v}$ given the components of $\mathbf{u}$ and $\mathbf{v}$? What can you say about the dot product if (a) $\mathbf{u}$ and $\mathbf{v}$ are parallel? (b) $\mathbf{u}$ and $\mathbf{v}$ are perpendicular? What does $\mathbf{v} \cdot \mathbf{v}$ give in terms of some geometric property of $\mathbf{v}$? How can you find the angle between $\mathbf{u}$ and $\mathbf{v}$ using the dot product? When is $\mathbf{u} \cdot \mathbf{v} = 0$? What happens when you swap the order of the terms in a dot product?

3. Define the component and projection of $\mathbf{u}$ onto $\mathbf{v}$ Which one is a scalar/vector? How do you calculate components/projections using the dot product? Given a diagram of $\mathbf{u}$ and $\mathbf{v}$, draw the projection of one to the other. What can you say about components/projections of $\mathbf{u}$ onto $\mathbf{v}$ if $\mathbf{u}$ and $\mathbf{v}$ are identical/parallel/perpendicular? What is the projection of a vector $\mathbf{v}$ in $\mathbb{R}^3$ to $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$? How can you recover $\mathbf{v}$ given these three projections? What are the components of $\mathbf{v}$ to $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$?

4. Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbb{R}^3$. Define the cross product $\mathbf{u} \times \mathbf{v}$ of two vectors $\mathbf{u}$ and $\mathbf{v}$. Is it a scalar or vector? How do you compute $\mathbf{u} \times \mathbf{v}$ given the components of $\mathbf{u}$ and $\mathbf{v}$? What is the direction of $\mathbf{u} \times \mathbf{v}$ in relation to $\mathbf{u}$ and $\mathbf{v}$? What does $|\mathbf{u} \times \mathbf{v}|$ represent geometrically? What can you say about the cross product if (a) $\mathbf{u}$ and $\mathbf{v}$ are parallel? (b) $\mathbf{u}$ and $\mathbf{v}$ are perpendicular? What is $\mathbf{v} \times \mathbf{v}$? When is $\mathbf{u} \times \mathbf{v} = 0$? Is it true that $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$? If not, what is the correct relationship? Find the cross products of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ with themselves, and with each other in both orders. Given two non-parallel vectors $\mathbf{u}$ and $\mathbf{v}$, how can you find a third vector perpendicular to both?

5. (a) Find the equation of a line given a point and parallel vector in (a) parametric vector form, (b) parametric Cartesion form, (c) symmetric form.

   (b) Given two points A and B, find the equation of the line passing through them. (Write this in terms of position vectors $\mathbf{a} = \overrightarrow{OA}$, $\mathbf{b} = \overrightarrow{OB}$, and a parameter t). How would you parameterise the line segment between A and B? How can you write the position vector of the midpoint of A and B in terms of $\mathbf{a}$ and $\mathbf{b}$? How would you find a point dividing the A and B in a given ratio?

   (c) How can you tell when two lines are parallel? How can you tell if they intersect? How do you find the angle between two intersecting lines?

   (d) How do you find the distance from a point to a line?

6. (a) Find the equation of a plane given a point and a normal vector. What is a normal vector to the plane given by $ax+by+cz = d$? Find the equation of a line orthogonal to the plane through a given point.

   (b) How can you tell if two planes are parallel? What about perpendicular? How do you find the angle between two intersecting planes?

(c) How do you find the equation of the line of intersection of two planes? (Hint: Use the normal vectors of the planes to find a vector parallel to the line. Then find one point lying on the line by finding a solution to the equations of the planes.)

(d) Given three non-collinear points A, B, and C in $\mathbb{R}^3$, find the equation of a plane passing through A, B, and C. (Hint: Take the cross product of $\overrightarrow{AB}$ and $\overrightarrow{AC}$). Can you still apply this method if A, B, and C lie on a straight line? Explain why geometrically. How can you calculate the area of the triangle spanned by A, B, and C?

(e) How do you find the distance of a point to a plane? What about the distance between two parallel planes?

7. (a) What is a vector function (say, in $\mathbb{R}^3$)? What is the natural domain of a vector function? How do you take limits of vector functions. Define the derivative of a vector fundtion. What does is mean for a vector function to be continuous/differentiable? How do you compute the derivative of a vector function given its component functions?

   (b) Suppose C is a curve parameterised by a vector function $\mathbf{r}(t)$, $a \leq t \leq b$. What are the initial/terminal points of the curve? What does the derivative $\mathbf{r}'(t)$ represent geometrically? What is a unit tangent vector to a curve, and how would you find it?

   (c) How do you find the equation of a tangent line to a curve at $\mathbf{r}(t_0)$? (Hint: How would you find a tangent vector at $\mathbf{r}(t_0)$? Then use the standard method for finding equations for lines given a point and a vector).

   (d) If $\mathbf{r}(t)$ represents position at time t, what do $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$ respresent? What does $|\mathbf{r}'(t)|$ respresent?

   (e) Suppose you have a curve in $\mathbb{R}^2$ given by parametric equations for x and y in terms of t. Eliminate t to obtain an equation involving only x and y, and draw a graph of this equation. Must the parametrised curve run over the entire graph?

   (f) Find a parameterisation of a circle with radius R and centre $(h.k)$ in $\mathbb{R}^2$. How would you indicate that you only want the top half of the circle? What if you wanted to reverse the direction of the path?

   (g) Find a parameterisation of the intersection of two surfaces in $\mathbb{R}^3$ (note: only for surfaces with simple equations that allow you to draw their projections to the co-ordinate planes).

   (h) How can you tell when two moving objects collide, given vector equations for their paths of motion. How can you tell when the underlying curves intersect?

8. (a) Write (short) proofs of basic geometric statements using properties of vectors.

   (b) Prove basic formulae involving vector functions (such as differentiation rules) by doing calculations for component function.