11. Homework

Show your work! Justify your answers!

Read Sec. 9.1 - 9.5
Work the problems
Sec. 9.1 #'s : 17, 23,
Sec. 9.3 #'s : 6, 17.
Sec. 9.5 #'s : 3, 17.

1) Show that
\[ \int_{-\pi}^{\pi} \cos(nx) \sin(mx) \, dx = 0, \] for integers \( n, m \).

Also show that
\[ \int_{0}^{L} \sin\left(\frac{n\pi}{L} x\right) \sin\left(\frac{m\pi}{L} x\right) \, dx = 0, \] for integers \( n \neq m \).

2) On the interval \((0, 2)\), find the Fourier Sine series for \( f = e^x \).

3) Solve
\[
\begin{aligned}
&u_t - u_{xx} = 0, & 0 < x < 2, & t > 0, \\
&u(x, 0) = e^x, \\
&u(0, t) = 0, & u(2, t) = 0.
\end{aligned}
\]

4) Solve
\[
\begin{aligned}
&u_t - u_{xx} = 0, & -1 < x < 1, & t > 0, \\
&u(x, 0) = x, \\
&u(-1, t) = u(1, t), \\
&u_x(-1, t) = u_x(1, t).
\end{aligned}
\]