Sec. 3.2, #’s 2, 8: a), b); 15.
Sec. 4.2 # 1.

and

1) Let \( \phi_1 \equiv 1 \), \( \phi_2 = x \), \( \phi_3 = x^2 \), defined on the interval \((-1, 1)\). With respect to the inner product

\[
< \phi, \psi >_{1-x^2} = \int_{-1}^{1} \phi \psi (1 - x^2) \, dx,
\]

find linear combinations \( \psi_1, \psi_2, \psi_3 \), of \( \phi_1, \phi_2, \phi_3 \) such that the linear combinations form a set of orthonormal functions.

2) Find the best approximation of \( f(x) = \sin \pi x \) with respect to the norm

\[
\| \phi \|_{1-x^2}
\]

in \( \text{span} \{ \phi_1, \phi_2, \phi_3 \} \).

3) Verify directly Bessel’s inequality with regard of the best approximation you found in 2).

4) Find the eigenvalues and eigenfunctions for the Sturm-Liouville problem

\[
\begin{align*}
X'' + \lambda X &= 0, \quad \text{on } (0, l); \\
X(0) &= 0, \\
X'(l) &= 0.
\end{align*}
\]