4. Homework

Show all your work! Justify your answers!

Problems from the book.
Sec.1.1 # s 12, 13. Sec. 2.5 # 1. Sec. 2.4 # 3.

1) Determine whether the equations
$$u_t - iu_{xxx} = 0,$$
and
$$u_{tt} + iu_{xxx} = 0,$$
are diffusive, dispersive or neither of the two.

2) i) Find the solution $v(x,t,\tau)$ of
$$\begin{cases}
  u_{tt} - c^2 u_{xx} = 0, \\
  u(x,0;\tau) = f(x,\tau), \\
  u_t(x,0;\tau) = 0.
\end{cases}$$
Here $\tau$ is a fixed parameter.

ii) For $w(x,t) = \int_0^t v(x,t-\tau,\tau) d\tau$, calculate
$$w_{tt} - c^2 w_{xx}.$$

3) Find the general solution of the IBVP for the wave equation with homogeneous Neumann boundary condition:
$$\begin{cases}
  u_{tt} - c^2 u_{xx} = 0, \\
  u_x(0,t) = 0, \\
  u(x,0) = \phi(x), \\
  u_t(x,0) = \psi(x).
\end{cases}$$

4) Let $\phi(x)$ be a function defined on the interval $(-\infty,a]$, with $\phi(a) = 0$. Define on $(-\infty,\infty)$ the function $\hat{\phi}$ by
$$\hat{\phi}(x) = \begin{cases}
  \phi(x) & \text{for } x \in (-\infty,a], \\
  -\phi(2a-x) & \text{for } x \in [a,\infty),
\end{cases}$$
Verify that $\hat{\phi}(a+x) = -\hat{\phi}(a-x)$. Use this to solve
$$\begin{cases}
  u_{tt} - c^2 u_{xx} = 0, \\
  u(a,t) = 0, \\
  u(x,0) = \phi(x), \\
  u_t(x,0) = 0.
\end{cases}$$