CALCULUS III FALL 1999

REVIEW 1

Reason is the slow and tortuous method by which those who do not know the truth discover it.

BLAISE PASCAL

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1. Find the equation of the tangent to the curve $x = 2\sec t,\ y = 2\tan t$ at the point where $t = -\pi/6$.

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2. Find the area of the region between the curve $x = e^{2t},\ y = e^{-t}$ and the x-axis from $t = 0$ to $t = \ln 5$.

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3. The path of a projectile fired from ground level with speed v_0 feet per second at an angle α with the ground, is given by the parametric equations $x = (v_0 \cos\alpha)t,\ y = -16t^2 + (v_0 \sin\alpha)t$. Show that the path is a parabola. Find the time of flight. Show that the range is $(v_0^2/32)\sin 2\alpha$. For a given v_0, what value of α maximizes the range?

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4. Let P be a point of the cycloid corresponding to the parameter value θ. Let C be the circle with center $(r\theta, r)$ and radius r. Prove that the tangent to the cycloid at the point P passes through the highest point of C. Where does the normal intersect this circle?

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5. Assume a circle of radius $r/4$ roll inside a fixed circle of radius r. Find the parametric equations of the curve traced out by a point P on the circumference of the rolling circle.

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6. Assume a circle of radius b rolls on the outside of a fixed circle of radius a. Find the parametric equations of the curve traced out by a point on the perimeter of the rolling circle.

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7. Find the slope of the tangent to the graph of $r = 3 + 3\cos\theta$ at the point where $\theta = \pi/6$.

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8. Find the points of intersection of the curves $r = 5\sin\theta$ and $r = 2 + \sin\theta$. Find the area of the region inside the limaçon and outside the circle.

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9. A racing car on a elliptical track $x^2/400 + y^2/100 = 1$ went out of control at the point $(16, 6)$ and thereafter continued on the tangent line until hit a tree at $(14, k)$. What is k?

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10. At what points does the curve $x = 2a\cos t - a\cos 2t,\ 2a\sin t - a\sin 2t$ have vertical or horizontal tangents? Find the area enclosed by this curve.

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11. Find the points of intersection of the curves $r^2 = 4\cos\theta$ and $r = 3 + \cos\theta$. Do the same for the curves $r = \sin 2\theta$ and $r = \sin\theta$.

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12. A particle moves from time $t = 0$ to $t = 1$ along the curve $x = 4t - \sin\pi t,\ y = 4t + \cos\pi t$. When does the particle have minimum speed? When does it have maximum speed? What is the area under the curved path? What is the slope of the tangent line at the point $(1 - 1/\sqrt{2}, 1 + 1/\sqrt{2})$?