6. \[
\frac{1}{\sqrt{2 + x}} = \frac{1}{\sqrt{2}} \left(1 + \frac{x}{2}\right)^{-1/2} = \frac{1}{\sqrt{2}} \sum_{n=0}^{\infty} \frac{(-1/2)(-3/2) \cdots (-1/2 - n + 1)}{n!} \left(\frac{x}{2}\right)^n
\]
\[
= \frac{1}{\sqrt{2}} \sum_{n=0}^{\infty} (-1)^n \frac{1 \cdot 3 \cdots (2n-1)}{2^{2n} n!} \frac{x^n}{2^n} = \frac{1}{\sqrt{2}} \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{2^{3n} (n!)^2} \frac{x^n}{2^n}.
\]
We applied the binomial theorem to the variable \(x/2\) so the possible range of values is given by \(|x/2| < 1\) which gives \(|x| < 2\) and so the radius of convergence is 2.

8. \[
\frac{x^2}{\sqrt{1 - x^3}} = x^2 (1 - x^3)^{-1/2} = x^2 \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{2^{2n} (n!)^2} (-x^3)^n
\]
\[
= \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n} (n!)^2} x^{3n+2}.
\]
We applied the binomial theorem to the variable \(-x^3\) so the possible range of values is given by \(|-x^3| < 1\) which is the same as \(|x| < 1\) and so the radius of convergence is 1.
12. 

\[(4 + x)^{3/2} = 8 \left(1 + \frac{x}{4}\right)^{3/2} = 8 \sum_{n=0}^{\infty} \frac{\left(\frac{3}{2}\right)\left(-\frac{1}{2}\right) \cdots \left(\frac{3}{2} - n + 1\right)}{n!} \left(\frac{x}{4}\right)^n\]

\[= 24 \sum_{n=0}^{\infty} (-1)^n 1 \cdots (2n - 5) \frac{(2n - 4)!}{2^{2n} n!} \left(\frac{x}{4}\right)^n = 24 \sum_{n=0}^{\infty} \frac{(2n - 4)!}{2^{2n - 2} n!(n - 2)!} x^n.\]

It follows that the Taylor polynomials are

\[T_1(x) = 8 \left(1 + \frac{3x}{8}\right) = 8 + 3x,\]

\[T_2(x) = 8 \left(1 + \frac{3x}{8} + \frac{\left(\frac{3}{2}\right)\left(-\frac{1}{2}\right)}{2}\right) \frac{x^2}{16} = 8 + 3x + \frac{3x^2}{16},\]

\[T_3(x) = 8 \left(1 + \frac{3x}{8} + \frac{\left(\frac{3}{2}\right)\left(-\frac{1}{2}\right)}{2} + \frac{\left(\frac{3}{2}\right)\left(-\frac{1}{2}\right)\frac{\left(\frac{3}{2}\right)\left(-\frac{1}{2}\right)}{3!}}{64}\right) \frac{x^2}{16} = 8 + 3x + \frac{3x^2}{16} - \frac{x^3}{128}.\]

We applied the binomial theorem to the variable \(x/4\) so the possible range of values is given by \(|x/4| < 1\) which gives \(|x| < 4\) and so the radius of convergence is 4. The graphs are given below
16. 

\[(8 + x)^{1/3} = 2 \left(1 + \frac{x}{8}\right)^{1/3} = 2 \sum_{n=0}^{\infty} \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\cdots\left(\frac{1}{3} - n + 1\right)}{n!} \frac{x^n}{8^n} \]

\[= 2 \sum_{n=0}^{\infty} (-1)^{n-1} \frac{2.5 \cdots (3n - 4)}{2^{3n} 3^n n!} x^n.\]

We applied the binomial theorem to the variable \(x/8\) so the possible range of values is given by \(|x/8| < 1\) which gives \(|x| < 8\) and so the radius of convergence is 8.

For part b), we put \(x = 0.2\) to get

\[2 \sum_{n=0}^{\infty} (-1)^{n-1} \frac{2.5 \cdots (3n - 4)}{2^{3n} 3^n n!} \frac{1}{5^n}.\]

The alternating nature of this series suggests that the required answer is given by the partial sum \(s_n\) where

\[\frac{2.5 \cdots (3n - 1)}{2^{3n+3} 3^n n!} \frac{1}{5^{n+1}} < 0.00005.\]

We find that this is satisfied when \(n = 2\) and so the required estimate is

\[\sqrt[3]{8.2} \approx 2 \left(1 + \frac{1}{120} - \frac{1}{14400}\right) \approx 2.0165.\]

20. 

\[(1 + x^3)^{-1/2} = \sum_{n=0}^{\infty} \frac{\left(-\frac{1}{3}\right)\left(-\frac{3}{3}\right)\cdots\left(-\frac{1}{3} - n + 1\right)}{n!} (x^3)^n\]

\[= \sum_{n=0}^{\infty} (-1)^n \frac{1.3\cdots(2n - 1)}{2^n n!} x^{3n} = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{2^{2n} (n!)^2} x^{3n}.\]

It follows that

\[\frac{f^{(3n)}(0)}{(3n)!} = (-1)^n \frac{(2n)!}{2^{2n} (n!)^2} \quad \text{and so} \quad f^{(9)}(0) = -9! \frac{6!}{2^6 r} = -113,400.\]
10. We have \( f'(x) = -1/x^2 \), \( f''(x) = 2/x^3 \) and, in general, \( f^{(n)}(x) = (-1)^n n! / x^{n+1} \), which gives \( f^{(n)}(1) = (-1)^n n! \). It follows that the required Taylor series is

\[
\sum_{n=0}^{\infty} (-1)^n (x - 1)^n.
\]

Notice that there is another way to reach this same conclusion,

\[
\frac{1}{x} = \frac{1}{1 - (1 - x)} = \sum_{n=0}^{\infty} (1 - x)^n = \sum_{n=0}^{\infty} (-1)^n (x - 1)^n.
\]

It follows that \( T_1(x) = 1 - (x-1) = 2-x \), \( T_2(x) = 1 - (x-1) + (x-1)^2 = 3 - 3x + x^2 \) and \( T_3(x) = 1 - (x-1) + (x-1)^2 - (x-1)^3 = 4 - 6x + 4x^2 - x^3 \). The graphs are shown below.
22. We have \( f^{(n)}(x) = (-1)^{n-1}(n-1)!/x^n \) if \( n \geq 1 \). It follows that

\[
\ln x = \ln 4 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n4^n} (x - 4)^n.
\]

The third Taylor polynomial is

\[
T_3(x) = \ln 4 + \frac{x - 4}{4} - \frac{(x - 4)^2}{32} + \frac{(x - 4)^3}{192}.
\]

The remainder term is \( R_3(x) = -(x - 4)^4/(4z^4) \) where \( z \) is between \( x \) and 4. We have \( 3 \leq x \leq 5 \) and so \( 3 < z < 5 \), hence \( |R_3(x)| < 1/(4(3^4)) = 1/324 \approx 0.0031 \). The graph of \( f \) and \( T_3 \) are given below.