MATH 4093/5093 – Applied Numerical Methods

Catalog description: Prerequisite: 2443, 3113 or 3413; 3333 or 4373, or permission of the instructor. Numerical treatment of ordinary differential equations, numerical linear algebra and applications, basic numerical methods for partial differential equations. (Alt. Sp.)

Textbook: B. Bradie. *A Friendly Introduction to Numerical Analysis.* Pearson/Prentice Hall, 2006, ISBN 0-13-013054-0. The course will cover significant parts of Chapters 3, 4, 7–11.

Office hours: Monday 1:30–2:45 p.m., Thursday 9:15–10:30 a.m., or by appointment.

Prerequisites: *Multivariable Calculus* at the level of MATH 2443, *Ordinary Differential Equations* at the level of MATH 3113 or MATH 3413, *Linear Algebra* at the level of MATH 3333, or permission of the instructor. Previous knowledge of *Numerical Analysis* is *not* assumed, but courses like MATH 4073 or Electrical Engineering 3793 will be useful complements to this course.

Description: Modern applied sciences rely heavily on solving problems of linear algebra and (ordinary and partial) differential equations. In most cases these problems do not have a simple explicit solution, and can only be solved numerically. Understanding the mathematical ideas behind the methods for numerical computations is vitally important for anybody using numerical methods.

This course is an introduction to some of the most important numerical methods for treating linear algebra problems (solving systems of linear equations, finding eigenvalues, etc.), for solving initial and boundary value-problems for ordinary differential equations as well as some important for practice partial differential equations (Poisson, heat, and wave equations). Many problems discussed in class and given as a homework will be motivated by physical and engineering applications.

Knowledge of a programming language is *not* a prerequisite for taking the course. Some homework problems will require simple programming in MATLAB, but most programs will be written by the instructor and the students will only need to run them and explain the behavior of the numerical method from a mathematical point of view.

Grading: The grade for the course will be based on regularly assigned homework, two midterm exams and a final exam. In addition, students enrolled in MATH 5093 should anticipate to be assigned some additional more difficult problems in most of the homework sets and in the exams, which will constitute the graduate-level credit portion of the course. The midterms will be given during the fifth and the tenth weeks of the semester. The approximate contributions of each type of assignment to the final grade are: homework 30 %, midterm exams 20 % each, final exam 30 %.

Reasonable Accommodation Policy: The University of Oklahoma is committed to providing reasonable accommodation for all students with disabilities. Students with disabilities who require accommodations in this course are requested to speak with the instructor as early in the semester as possible. Students with disabilities must be registered with the Office of Disability Services prior to receiving accommodations in this course. The Office of Disability Services is located in Goddard Health Center, Suite 166, phone 405-325-3852 or TDD (only) 405-325-4173.

Codes of Behavior: All cases of suspected academic misconduct will be referred to the Dean of the College of Arts and Sciences for prosecution under the University's Academic Misconduct Code. More details on the University's policies concerning academic misconduct can be found at `http://www.ou.edu/provost/integrity/`. For information on your rights to appeal charges of academic misconduct see `http://www.ou.edu/provost/integrity-rights/`. Students are also bound by the provisions of the University of Oklahoma Student Code, which can be found at `http://judicial.ou.edu/content/view/27/32/`.

Main topics:

- Numerical linear algebra.
- Numerical solutions of initial-value problems for ordinary differential equations.
- Numerical approximation of functions.
- Approximating eigenvalues.
- Numerical solutions of nonlinear systems of equations.
- Numerical solutions of boundary-value problems for ordinary differential equations.
- Numerical solutions of partial differential equations.

Lecture-by-lecture description:

Part I – ODEs

1. Initial-value problems for ODEs – elements of theory.
2. Euler's method, analysis of Euler's method.
3. Taylor methods, rate of convergence or Taylor methods.
4. Preparation for Runge-Kutta methods – Lagrange interpolation polynomial, numerical integration.
5. Second-order Runge-Kutta methods – general derivation, particular cases (modified Euler, Heun, optimal RK2).
6. Fourth-order Runge-Kutta methos.
7. Multistep methods – Adams-Bashforth method.
8. Multistep methods (cont.) – Adams-Moulton method, predictor-corector schemes.
9. Convergence and stability analysis of numerical methods for IVP for ODEs.
10. Systems of equations and higher-order equations.

11. Absolute stability and stiff equations.

<u>Part II – Linear Algebra</u>

12. Linear systems of equations – elements of theory. Gaussian elimination.
13. LU factorization.
14. Pivoting
15. Special classes of matrices.
16. Example of application: 2-point boundary-value problem for ODEs.
17. Vector and matrix norms.
18. Condition number, idea of iterative methods.
19. Iterative methods: Jacobi's method.
20. Iterative methods (cont.): Gauss-Seidel method.
21. The conjugate gradient method.
22. Eigenvalues, eigenvectors, characteristic polynomial, spectral radius.
23. Analysis of iterative methods, successive over-relaxation.
24. Numerical methods for eigenvalues and eigenvectors, the power method.
25. The inverse power method, examples of application.
26. Basic ideas of the QR algorithm.

<u>Part III – Applications of Numerical Linear Algebra</u>

27. Nonlinear systems of equations.
28. Cubic spline interpolation.
29. Two-point BVPs for ODEs.

<u>Part IV – Numerical Methods for PDEs</u>

30. Elliptic PDEs – basic facts and physical motivation, Poisson and Laplace equations.
31. Finite differences method for a Dirichlet BVP for Poisson equation on a rectangle.
32. Finite differences method for a Neumann BVP for Poisson equation on a rectangle.
33. Ideas for dealing with non-rectangular domains.
34. Parabolic PDEs – basic facts and physical motivation, heat equation.

35. The $(1+1)$-dimensional heat equation on a finite interval with Dirichlet boundary conditions.
36. Stability issues in the numerical treatment of the heat equation.
37. Numerical methods for more general parabolic equations.
38. Hyperbolic PDEs – basic facts and physical motivation, wave equation, advection and advection-diffusion equations.
39. Upwind differencing methods for advection equations, CFL condition.
40. Numerical solution of the wave equations.