Notations

General notations

$y := 5 + x$ means that we *define* y to be equal to $5 + x$; the two dots in ":=" are at the side of the object that is being defined.

$\forall$ means "for each", "for every", "for all".

$\exists$ means "there exist(s)"; $\exists!$ means "there exists a unique...".

s.t. means "such that".

Examples:

- $\forall x \geq 0\ \exists\, y$ s.t. $y^2 = x$ (Note that y is not unique, i.e., for $x = 25$, y may be 5 or -5.)
- $\forall x \neq 0\ \exists!\, y$ s.t. $xy = 1$ (Namely, $y = \frac{1}{x}$.)
- By definition, the sequence $a_1, a_2, \ldots$ of real numbers tends to a limit a (notation: $a_n \to_{n\to\infty} a$ or $\lim_{n\to\infty} a_n = a$) if $\forall\, \epsilon > 0\ \exists\, N \in \mathbb{N}$ s.t. $\forall\, n > N,\ |a_n - a| < \epsilon$.

Set-theoretic notations

$A = \{a_1, a_2, \ldots, a_n\}$ means that the set A consists of n elements, namely, $a_1, a_2, \ldots, a_n$.

Remark: In general, the elements of a set A are not naturally ordered.

$\{a\}$ is the *set* consisting of one element only (namely, the element a).

$|A|$ (or $\#A$) stands for the *cardinality* of A, i.e., the number of elements in the set A.

Important sets:

- the set of *integers*: $\mathbb{Z} := \{\ldots, -2, -1, 0, 1, 2, \ldots\}$
- the set of *natural numbers*: $\mathbb{N} := \{1, 2, 3, \ldots\}$
- the set of *real numbers* $\mathbb{R}$
- the set of *rational numbers*: $\mathbb{Q} := \{\frac{p}{q} : p \in \mathbb{Z},\ q \in \mathbb{N}\}$
- intervals of $\mathbb{R}$: $(a, b) := \{x \in \mathbb{R} : a < x < b\}$, $(a, b] := \{x \in \mathbb{R} : a < x \leq b\}$, $[a, b) := \{x \in \mathbb{R} : a \leq x < b\}$, $[a, b] := \{x \in \mathbb{R} : a \leq x \leq b\}$

B^A stands for the set of all maps (i.e., functions) f from A to B. Note that each element $a \in A$ must go to some $f(a) \in B$, while not all $b \in B$ are necessarily of the form $f(a)$ for some $a \in A$.

Exercise: Prove that for finite sets A and B, $|B^A| = |B|^{|A|}$, and that the number of subsets of the set A is $2^{|A|}$.

Remark: *Difference between* $\in$ *and* $\subset$

- $\omega \in A$ means that ω is an element of the set A.
- $A \subset B$ means that the set A is a subset of the set B (i.e., that each element in A also belongs to B).

Example: $5 \in \mathbb{N}$, but $\{5\} \subset \mathbb{N}$.

Logic notations

(P) $\Rightarrow$ (Q) means that the statement (P) implies the statement (Q).

(P) $\Leftrightarrow$ (Q) means that the statements (P) and (Q) are equivalent.

Examples:

- $(x \in \mathbb{N}) \;\Rightarrow\; (x^2 \in \mathbb{N})$ (Clearly, the converse – namely, that $x^2 \in \mathbb{N}$ implies $x \in \mathbb{N}$ – is false.)
- $(x > 0) \;\Leftrightarrow\; (x^3 > 0)$