Syllabus for MATH 4513, Senior Mathematics Seminar, Sec. 001

Fall 2015

Lectures: MWF 12:30 – 1:20 p.m., 120 PHSC

Class instructor: Prof. Nikola Petrov, `npetrov@ou.edu`, 802 PHSC, office phone 325-4316
Office Hours: Mon 2:30–3:30 p.m., Wed 11:20 a.m.–12:20 p.m., or by appointment, in 802 PHSC
Class web page: `http://www2.math.ou.edu/~npetrov/math4513_f15.html`

Prerequisites: MATH 2443 (Calculus IV) or MATH 2934 (Diff & Int Calculus III); MATH 2513 (Discrete Math Structures); MATH 3113 (ODEs) or MATH 3413 (Physical Math I); MATH 3333 (Linear Algebra I); and senior standing.

Texts: We may use parts of the following materials (all the books are freely available from the OU Libraries web-site for OU students):

1. M. W. Hirsch, S. Smale, R. L. Devaney. *Differential Equations, Dynamical Systems, and an Introduction to Chaos*. 3rd ed., Academic Press, 2012

2. B. Deconinck's notes on Dynamical Systems and Chaos, available for downloading at `http://depts.washington.edu/bdecon/bernard/` (click on 'Dynamical Systems (U)')

3. S. Sternberg's notes on Dynamical Systems, available for downloading at `http://www.math.harvard.edu/library/sternberg/text/book.pdf`

4. J. D. Meiss. *Differential Dynamical Systems*. Society for Industrial and Applied Mathematics, 2007

5. S. Wiggins. *Introduction to Applied Nonlinear Dynamical Systems and Chaos*. 2nd ed., Springer, 2003

6. L. Barreira, C. Valls. *Dynamical Systems*. Springer, 2013

7. J. Jost. *Dynamical Systems*. Springer, 2005

Grading: Your grade will be based on the following:

Homework	20 %
Quizzes	10 %
Attendance and class participation	5 %
Midterm exam	25 %
Project	40 %

Course objective: The main objective of this course is to give you an idea of how various concepts you have learned so far are inter-connected, and how they are used in non-trivial ways in other areas of mathematics and the sciences. We will mainly discuss problems from the theory of dynamical systems – an area of mathematics studying the behavior of systems that are so complicated that their exact solutions cannot be found. For example, since a nonlinear system of ODEs does not satisfy the Superposition Principle, finding the general solution of such a system is very complicated and often impossible.

Instead of trying to find the exact solution of a complicated system, one can instead try to understand the qualitative behavior of the solutions, i.e., are there constant solutions, are there periodic solutions, do all solutions stay in a bounded region or drift off to infinity, etc. We will derive nonlinear (systems of) ODEs occurring in some simple mechanical or biological problems, and will analyze the behavior of their solutions. We will develop some methods for studying their bifurcations, i.e., situations in which the solutions of the system change their qualitative behavior dramatically for a small change of some parameter. In particular, we will study saddle-node/transcritical/pitchfork/Hopf bifurcations, limit cycles, hysteresis, stability of periodic orbits, and related questions. We will discuss the famous Lorenz system and similar systems exhibiting highly irregular behavior ("chaos").

Another big topic we will cover will be the asymptotic behavior of highly iterated maps, i.e., the behavior of the composition $f \circ f \circ \cdots \circ f(x)$, where the function ("map") f is composed with itself n times, for $n \to \infty$. We will study bifurcations in some maps, their periodic orbits, Lyapunov exponents, and the universal behavior of such maps. If time permits, we will touch on different concepts of dimension that are important in the context of dynamical systems.

Since I will not follow very closely any textbook, it is very important that you make an effort to attend every class. Since this is not a class in which we are supposed to cover systematically certain material like in all your other classes, I will do my best to make the class exciting by cover interesting topics. At the same time I want to warn you that this class will be among the hardest math classes you will have taken when you graduate. The main reason is that we will be using concepts and techniques of most of the math classes you have taken so far, and much more. I want to emphasize more on understanding the concepts and the beauty of math and the power of its applications, rather than memorize formulas and theorems. I would like you to approach the course with an open mind, and to work very hard. Your success in the class is a result of your own efforts and willingness to learn. I will be happy to help you along the way.

Homework/quizzes: I will assign homework periodically. The homework assignments will be given on the class web site. Your homework solutions must be turned in at the beginning of class on the due date. Giving just an answer to a problem is not worthy any credit – you have to write a complete solution which gives your step-by-step reasoning and is written in grammatically correct English. Although good exposition takes time and effort, writing your thoughts carefully will greatly increase your understanding and retention of the material. *You are encouraged to discuss the homework problems with other students, but you should write up the solutions in your own words. Please use your best handwriting! (If I am cannot easily read what you have written, I will not read it and you will receive no credit for this problem.) No late homework will be accepted!*

Short in-class quizzes will be given throughout the semester; some of them will be announced in advance, others will be pop-quizzes. The lowest quiz grade will be dropped. *There will be no make-up quizzes except if you are away for a University mandated activity, or a family emergency, or if you can provide a doctor's note!*

Project: One of the major components of the course is a project, with a written report and a presentation. Every student has to do the project. The process of writing and presenting the project will consist of several steps; your performance for each step will be graded.

Step 1. Each project will be done by teams of two students. You should start right away looking for a team partner. No later than one month after the beginning of the semester I will suggest projects; each team will give me three choices in order of preference, so that we can avoid

two different groups working on the same project. If you have a different idea for a project topic, you can talk to me about it and we can discuss if it is good.

Step 2. Roughly a month after the project topics have been assigned to the teams, each member of each team should have a meeting with me to report your progress towards the project. I need every student to have the meeting with me even if your report is that there is nothing to report. Note that these mid-semester meetings will be with each individual student, not with the teams of two.

Step 3. Towards the end of the semester, you should meet with me and show me an initial draft of your project.

Step 4. During class time we will schedule presentations of the projects. A presentation will typically be 25 minutes (two presentations per lecture), and should be given by all members of the team. Attendance for all (non-presenting) students is mandatory.

Step 5. You should prepare your final project, taking into consideration the comments from your fellow students and myself. This has to be submitted by the day of the final exam.

Attendance: You are expected to attend all lectures and all discussion classes, and you are responsible for all information given out during them.

All electronic equipment should be turned off before the start of every lecture and discussion class, and should remain off until the class is dismissed. Since learning calculus requires your full attention, activities such as *conversing with other students, eating, sleeping, reading a newspaper, listening to headsets, using computers, cell phones, or other electronic devices, are not allowed!*

Use of calculators and technology: We will make use of the computer algebra system Mathematica, which is freely available on all OU computers. A basic calculator may be needed for a few of the homework problems, but use of electronic devices of any kind during exams is prohibited.

Some important dates:

(1) First day of classes: Monday, August 24, 2015.

(2) Labor day holiday (no classes): Monday, September 7, 2015.

(3) Last day to withdraw with an automatic W: Friday, October 30, 2015 for undergraduate students and Friday, October 2, 2015 for graduate students.

(4) Last day to withdraw without petition to the Dean: Friday, October 30, 2015 (for graduate students a W/F grade is assigned for withdrawals processed during the period October 5–October 30).

(5) OU-Texas football game holiday: Friday, October 9, 2015.

(6) Thanksgiving break (no classes): November 25–27, 2015.

(7) Last day of classes: Friday, December 11, 2015.

Policy on W/I grades: Through the end of the sixth week of the semester, students can withdraw from the course with an automatic W. Between the seventh and tenth weeks of the semester, undergraduate students can continue to withdraw with an automatic W, but graduate students must obtain the instructor's signature on the University's "drop form" to withdraw from

the course, and along with the signature the instructor must indicate whether the student is passing or failing at the time of the withdrawal. After the tenth week of the semester, all students can only withdraw via petition to the Dean of their college. The petition process also requires the instructor's signature with a passing-failing indication at the time the petition is filed. Note that a "failing" indication on the petition means that even if the petition is approved the grade in the course will be weighted in the GPA as an F.

The grade of I is not intended to serve as a benign substitute for the grade of F, and is only given if a student has completed the majority of the work in the course at a passing level (for example everything except the final exam), the course work cannot be completed because of compelling and verifiable problem beyond the student's control, and the student expresses a clear intention of making up the missed work as soon as possible. Moreover, current OU policies require that instructors and the affected students execute a written "Incomplete Contract" before a grade of I can be given. The contract makes clear: (1) what work is to be made up; (2) when the make-up work must be completed (which cannot be more than one calendar year from the assignment of the I); and (3) *what alternative grade will be assigned if the make-up work is not completed.* If the make-up work specified in the contract is not made up within one calendar year, then the alternative grade specified in the contract will be entered on the student's transcript. Thus the I grade does not became permanent on the transcript if it is not made up within one year.

Academic Misconduct: All cases of suspected academic misconduct will be referred to the Dean of the College of Arts and Sciences for prosecution under the OU Academic Misconduct Code. The penalties can be quite severe. *Don't do it!* For more details on the OU policies concerning academic misconduct see

`http://integrity.ou.edu/files/Academic_Misconduct_Code.pdf`

This link also has information about students' rights to appeal charges of academic misconduct. For information about admonitions (either accepting or contesting them) see

`http://integrity.ou.edu/files/Admonition.pdf`

Students are also bound by the provisions of the *OU Student Code*, which can be found at

`http://judicial.ou.edu/content/view/27/32/`

Students with disabilities: The University of Oklahoma is committed to providing reasonable accommodation for all students with disabilities. Students with disabilities who require accommodations in this course are requested to speak with the instructor as early in the semester as possible. Students with disabilities must be registered with the Office of Disability Services prior to receiving accommodations in this course. The Office of Disability Services is located in Goddard Health Center, Suite 166: phone 405–325–3852 or TDD (only) 405–325–4173.

Caminante, son tus huellas	Traveler, your footsteps
el camino y nada más;	are the road and nothing more;
caminante, no hay camino,	traveler, there is no road,
se hace camino al andar.	the road is made by walking.

From *Proverbios y Cantares*, Antonio Machado (1875–1939)