MATH 5483 Homework 9 Due Thu, Apr 10

Problem 1 (and only). Consider the case of the Haar multiresolution analysis, whose scaling filter is $h(0) = h(1) = \frac{1}{\sqrt{2}}$. You can use only the very definitions of the quantities to answer the questions below.

(a) Construct the function $m_0(\gamma)$ corresponding to the Haar scaling filter, and check that it satisfies the quadrature mirror filter coefficients.

(b) Find the wavelet filter $g(k)$, the wavelet $\psi(x)$, and the function $m_1(\gamma)$.

(c) Determine the explicit form of the approximation and detail operators, H and G, and their adjoints, H^* and G^*.

(d) Using the explicit expressions for the operators H, G, H^*, and G^*, check that the identities

$$H\,H^* = I\ , \qquad H\,G^* = I\ , \qquad H\,H^* + G\,G^* = I$$

are satisfied.

(e) Consider the finite (length 4) signal $c_0(0) = 36$, $c_0(1) = 12$, $c_0(2) = -18$, $c_0(3) = 6$, and think of its periodization (as in Section 8.3.2 of the book). Write the signal as a column vector

$$\mathbf{c}_0 = [\,36\ 12\ \ -18\ 6\,]^T\ .$$

Find the discrete wavelet transform,

$$\mathbf{d} = [\,\mathbf{d}_1\,|\,\mathbf{d}_2\,|\,\mathbf{c}_2\,]^T\ ,$$

of the signal $\mathbf{c}_0$, for the case of the Haar scaling filter. Lemma 8.22 from the book should be useful. Remember, you are allowed to use only the definitions!