MATH 3413   Homework 9   Due Thu, Apr 12, 2012

Sec. 9.2: problems 12, 17, 24(b).

Hint for Problem 9.2/12: By using that

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \ ,$$

one can derive the relation

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right] \ ,$$

which is useful in finding integrals of the form $\int \sin at \cos bt \, \mathrm{d}t$.

Hint for Problem 9.2/17: You may use the integrals

$$\int t\sin at \, \mathrm{d}t = \frac{1}{a^2}\left(\sin at - at\cos at\right) \ , \qquad \int t\cos at \, \mathrm{d}t = \frac{1}{a^2}\left(\cos at + at\sin at\right) \ .$$

Sec. 9.3: problems 2, 17, 18, 19.

Hint for Problem 9.3/2: The hint for Problem 9.2/17 will be useful.

Hint for Problem 9.3/18: This problem illustrates the dangers in differentiating a Fourier series termwise (i.e., term by term). Differentiate the Fourier series of t^2 on $t \in (0,2)$ given in the problem term by term. Compare your result with the Fourier series of the function $2t$ on $t \in (0,2)$ which can be easily obtained from your result in Problem 9.2/17. Discuss your result. Which condition from Theorem 1 on page 601 was violated?

Hint for Problem 9.3/19: Use Theorem 2 on page 605.

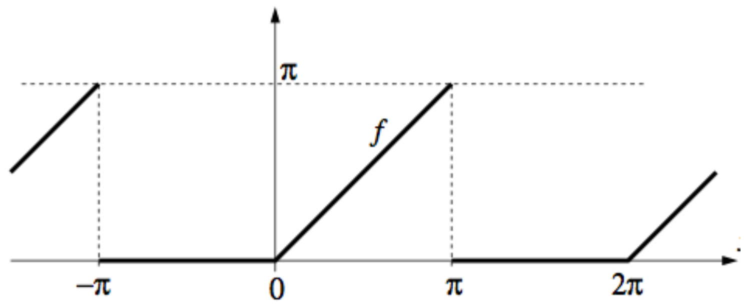
Sec. 9.4: problem 1.

Hint: The Fourier series for $F(t)$ can be easily found from the result in Example 1 of Section 9.1 (page 585).

Additional problem 1. Let f be a periodic function of period 2π which for t between $-\pi$ and π is defined as

$$f(t) = \begin{cases} 0 \ , & -\pi < t \le 0 \\ t \ , & 0 < t \le \pi \ ; \end{cases}$$

the graph of f is sketched in the figure below.



The Fourier series of f is the following (you do <u>not</u> have to prove this!):

$$f(t) = \frac{\pi}{4} - \frac{2}{\pi}\left(\cos t + \frac{\cos 3t}{3^2} + \frac{\cos 5t}{5^2} + \frac{\cos 7t}{7^2} + \cdots\right)$$
$$+\sin t - \frac{\sin 2t}{2} + \frac{\sin 3t}{3} - \frac{\sin 4t}{4} + \cdots .$$

Let the function g be a periodic function of period 2π sketched in the figure below. Write $g(t)$ in terms of $f(t)$. Using the Fourier series of f, find the Fourier series of g.