MATH 3413 Homework 9 Due Thu, Nov 2

Sec. 8.1: problems 6, 13, 22.

Sec. 8.2: problem 33 – solve part (a) completely; for part (b) just show that for integer α one indeed obtains polynomials, and find the explicit expressions for $H_0(x)$, $H_1(x)$, $H_2(x)$, and $H_3(x)$. Hermite polynomials occur in solving the harmonic oscillator problem in quantum mechanics.

Additional problem 1. For a smooth function f of one variable, the Taylor expansion around x_0 is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(x_0)\,(x - x_0)^n \ .$$

For a smooth function u of two variables, the Taylor expansion around (x_0, y_0) is given by

$$u(x,y) = \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{k=0}^{n} \binom{n}{k} \left.\frac{\partial^n u}{\partial x^k\, \partial y^{n-k}}\right|_{(x_0,y_0)} (x - x_0)^k\, (y - y_0)^{n-k} \ ,$$

where, by definition, $0! = 1$ and $\binom{0}{0} = 1$, and $\binom{n}{k}$ ("n choose k") are the binomial coefficients,

$$\binom{n}{k} = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!} \ .$$

Let $u(x,y) = (1+y)^2 + \sin(x+5y)$. Find the Taylor expansion of $u(x,y)$ around the point $(x_0, y_0) = (\frac{\pi}{2}, 0)$ up to (and including) the terms with $n = 2$. Here is the answer you have to obtain (you have to show your computations in detail):

$$u(x,y) = 2 + 2y - \tfrac{1}{2}\left(x - \tfrac{\pi}{2}\right)^2 - 5\left(x - \tfrac{\pi}{2}\right) y - \tfrac{23}{2}\, y^2 + (\text{higher order terms}) \ .$$

Additional problem 2. If $u(x,y)$ is a smooth function of two variables, find the following expressions:

(a) $\dfrac{d}{dt}\, u(t, \sin(3t))$; (b) $\dfrac{d}{dt}\, u(e^t, \sin(3t))$; (c) $\dfrac{d^2}{dt^2}\, u(e^t, \sin(3t))$.

Additional problem 3. Let $f(x)$ be a smooth function of one variable. Find the following expressions (where x and y are independent variables):

(a) $\dfrac{d}{dt}\, f(t^3)$; (b) $\dfrac{\partial}{\partial x}\, f(5x - 3y)$; (c) $\dfrac{\partial}{\partial y}\, f(5x - 3y)$;

(d) $\left.\dfrac{\partial}{\partial y}\, f(5x - 3y)\right|_{(x,y)=(2,7)}$.