MATH 5483 Homework 8 Due Thu, Apr 3

Problem 1.

(a) Use the definition of an adjoint operator, $\langle Gc, d\rangle = \langle c, G^* d\rangle$, to show that the adjoint of the detail operator G defined as

$$(Gc)(k) = \sum_n c(n)\,\overline{g(n-2k)}$$

is given by

$$(G^*c)(k) = \sum_n c(n)\, g(k-2n)\ .$$

(b) Directly from the definitions of the detail operator G, the upsampling and downsampling operators, and the convolution of signals, show that, for any signal $c(k)$,

$$Gc = \downarrow (c * \underline{g})\ , \qquad G^*c = (\uparrow c) * g\ ,$$

where the signal $\underline{g}(k)$ is defined by $\underline{g}(k) = \overline{g(-k)}$.

(c) Prove the properties

$$\widehat{(\tau_m c)}(\gamma) = e^{-2\pi i m \gamma}\,\widehat{c}(\gamma)\ , \qquad \widehat{(\uparrow c)}(\gamma) = \widehat{c}(2\gamma)\ .$$

Problem 2.

(a) Prove that if $h(k)$ is any filter and $g(k) = (-1)^k\,\overline{g(1-k)}$, then

$$\sum_n g(n) = 0 \quad \text{if and only if } \sum_n h(2n) = \sum_n h(2n+1)\ .$$

(b) Show that, for any signal $c(k)$, the following identities hold:

$$\widehat{(HH^*c)}(\gamma) = \widehat{(GG^*c)}(\gamma) = \left(\left| m_0\left(\frac{\gamma}{2}\right)\right|^2 + \left| m_0\left(\frac{\gamma}{2}+\frac{1}{2}\right)\right|^2 \right) \widehat{c}(\gamma)\ .$$

Please point out explicitly what properties you have used at each step of your derivation.