MATH 4433 Homework 7 Due 10/16/2014

Section 4.1: Exercises 7(d,e,f), 11, 14, 16(a,b). Hints and remarks:

- in Exercises 7(d) and 7(f), see the hints in the book; state explicitly which results you are using;
- one way to solve Exercise 7(e) is to notice that, for $n \geq 3$,

$$\frac{n^2}{n!} = \frac{n^2}{n(n-1)(n-2)(n-3)\cdots 2\cdot 1} \leq \frac{n}{(n-1)(n-2)} = \frac{n}{n^2-3n+2} \leq \cdots .$$

Section 4.2: Exercises 5(d,h,l), 6, 13, 15(b), 17(a). Hints and remarks:

- Exercise 5(d) follows easily from Exercise 4.1/7(f); in Exercise 5(h) you may use the ratio test for $L > 1$ (proved in Exercise 4.2/16), namely, fact that if $s_n > 0\ \forall n \in \mathbb{N}$ and $\lim(s_{n+1}/s_n) = L > 1$, then $\lim s_n = +\infty$ (compare this with Theorem 4.2.7); Exercise 5(l) is the same as Exercise 4.1/7(e), but here you have to solve it by using the ratio test (Theorem 4.2.7);
- only part (c) of Exercise 6 is true (and it follows easily from a theorem from Sec. 4.2);
- in Exercise 17(a), for $k > 0$ use the ratio test; for $k < 0$, use the result for $k > 0$ and Theorem 4.1.8.

Section 4.3: Exercises 3(a), 5, 7. Hints and remarks:

- Exercise 7 is similar to Example 4.3.4.

Food for Thought:

- Sec. **4.1**, exercises 6(b), 13, 15.
- Sec. **4.2**, exercises 1, 2, 7, 8, 15(a), 17(b), 19.
- Sec. **4.3**, exercises 1, 2(a,b), 14.

  Remark: The proof of Exercise 14 is long and technical, so you can skip it, but the result is fundamental – namely, in this exercise the existence of $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ is proved, and this limit is the base $\mathrm{e} = 2.7182818\ldots$ for natural logarithms.