MATH 3413 Homework 7 Due Fri, Oct 22, 2010

Sec. 7.2: problem 25.

Sec. 7.3: problems 3, 6, 8, 14, 17, 27, 30.

Sec. 7.4: problems 3, 7, 22, 23, 29, 36.
Hint to Problem 7.4/29: To find the Laplace transform of the product $tx'(t)$, use Theorem 2 on page 476 and the Corollary on page 454 to obtain the following:

$$\mathcal{L}\{tx'(t)\}(s) = -\mathcal{L}\{-tx'(t)\}(s) = -\frac{\mathrm{d}}{\mathrm{d}s}\mathcal{L}\{x'\}(s) = -\frac{\mathrm{d}}{\mathrm{d}s}[sX(s) - x(0)] \ ,$$

and similarly for $tx''(t)$; see also Example 5 on page 477.

Sec. 7.5: problems 2, 5, 25.

Additional problem 1.
Find the Laplace transform of the function $g(t) = t^2 \mathrm{e}^{-5t}$ in two ways:

(a) by using Theorem 1 from Section 7.3;

(b) by using Theorem 2 from Section 7.4.

You are allowed to use the table of Laplace transforms on page 446.

Additional problem 2.

(a) Find the general solution $\mathbf{X}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ of the first order system

$$\mathbf{X}'(t) = \begin{pmatrix} 0 & 1 \\ 6 & -1 \end{pmatrix} \cdot \mathbf{X}(t)$$

by first converting it to a second order differential equation.

(b) Find the solution of the initial value problem

$$\mathbf{X}'(t) = \begin{pmatrix} 0 & 1 \\ 6 & -1 \end{pmatrix} \cdot \mathbf{X}(t) \ , \qquad \mathbf{X}(0) = \mathbf{X}^{(0)} = \begin{pmatrix} x_1^{(0)} \\ x_2^{(0)} \end{pmatrix} \ . \tag{1}$$

(c) For most initial conditions $\mathbf{X}^{(0)}$ the solution $\mathbf{X}(t)$ of the initial value problem (1) will go to infinity. However, there is a line in the (x_1, x_2)-plane such that, $\mathbf{X}^{(0)}$ belongs to this line, $\mathbf{X}(t)$ will stay bounded for all t. Write down the equation of this line.