MATH 3413 Homework 6 Due Fri, Oct 15, 2010

Sec. 7.1: problems 3, 8, 14, 19, 25, 28, 38 (after you find the solution of this problem for general a and b, find it for $a = 1$, $b = 2$ and compare with your result in problem 7.1/8).

Sec. 7.2: problems 2, 5, 17, 19 (solve problems 17 and 19 both by applying Theorem 2 and by using the method of partial fractions).

Additional problem 1.

(a) Write the second-order linear ODE $x'' + 25x = 0$ as a first-order system. Rewrite this system in the form

$$\mathbf{X}'(t) = \mathsf{A} \cdot \mathbf{X}(t) \ ,$$

where $\mathbf{X}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ is the unknown vector-function, $\mathbf{X}'(t) = \begin{pmatrix} x_1'(t) \\ x_2'(t) \end{pmatrix}$ is its derivative, $\mathsf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ is the matrix of the coefficients of the first-order system, and the dot stands for the matrix-vector multiplication defined by

$$\mathsf{A} \cdot \mathbf{X}(t) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} a_{11}x_1(t) + a_{12}x_2(t) \\ a_{21}x_1(t) + a_{22}x_2(t) \end{pmatrix} \ .$$

Solve the ODE and then write its solution as a solution of the first-order system. Check that it is indeed a solution. Prove that the point $(x_1(t), x_2(t))$ moves on an ellipse in the (x_1, x_2)-plane as t changes. Find the semi-axes of this ellipse. Sketch by hand this ellipse in the (x_1, x_2)-plane.

(b) Write the second-order linear ODE $x'' + 6x' + 25x = 0$ as a first-order system. Rewrite this system in the form

$$\mathbf{X}'(t) = \mathsf{A} \cdot \mathbf{X}(t) \ .$$

Solve the ODE and then write its solution as a solution of the first-order system. Sketch by hand the solution of the second-order system as a curve in the (x_1, x_2)-plane. How does the point $(x_1(t), x_2(t))$ behave as $t \to \infty$?