MATH 3423 Homework 5 Due Thu, 10/03/19

Problem 1. Let C be the circle of radius R centered at the origin, traversed in positive (i.e., counterclockwise) direction.

(a) Directly from the definition of a contour integral in $\mathbb{C}$, compute the value of $\oint_C z^* \, \mathrm{d}z$.

Hint: The most efficient way of parameterizing C is $z(\theta) = R\mathrm{e}^{\mathrm{i}\theta}$, where the parameter θ varies from 0 to 2π.

(b) Based solely on your result from part (a), can you say for sure whether the function z^* is analytic in the domain surrounded by C? How did you came to your conclusion?

Problem 2.

(a) Use one of the results given on page 883 to give an upper bound for $\left| \int_C \frac{\mathrm{d}z}{z^2} \right|$, where C is the straight line segment from $2\mathrm{i}$ to $3 + 5\mathrm{i}$.

(b) Use that $\frac{\mathrm{d}}{\mathrm{d}z} \frac{1}{z} = -\frac{1}{z^2}$ to compute the exact value of $\left| \int_C \frac{\mathrm{d}z}{z^2} \right|$, and then compare it with the upper bound from part (a).

Problem 3. Evaluate $\oint_C \frac{z \, \mathrm{e}^{-z}}{z^3 + 8} \, \mathrm{d}z$, where C is the unit circle centered at the origin and traversed in positive direction.

Problem 4.

(a) Directly from the definition of a contour integral in $\mathbb{C}$, $\int_C f(z) \, \mathrm{d}z = \int_a^b f(z(t)) \, z'(t) \, \mathrm{d}t$ (where $z(t)$, $t \in [a, b]$ is a parameterization of the contour C), calculate the integral $\int_C z \, \mathrm{d}z$, where the contour C is given by the parametric equation $z(t) = 1 + \mathrm{e}^{\mathrm{i}t}$, and the parameter t varies from 0 to $\frac{\pi}{4}$.

(b) A much faster way to compute the integral from part (a) is to use the Theorem on page 888 of the book. Compute the value of the integral, and compare your result with the result from part (a). Did you use the specific form of the contour C in your calculation in this part of the problem?

(c) Evaluate the value of the integral $\int_C \left(z^6 + z\mathrm{e}^{z^2} \right) \mathrm{d}z$, where the contour C is a segment of a straight line from starting at 2 and ending at i.

Problem 5. Use Cauchy's Integral Formula to evaluate $\oint_C \frac{\tan z}{4z-\pi}\,\mathrm{d}z$, where C is the contour (traversed in positive direction) given by

(a) $|z| = 1$;

(b) $|z| = \frac{1}{2}$.

Problem 6. Evaluate $\oint_C \frac{\mathrm{e}^{\mathrm{i}z}}{z^2+1}\,\mathrm{d}z$, where C is the contour given by $|z| = 2$ (traversed in positive direction).

Problem 7. Evaluate $\oint_C \frac{\cos z}{z(z-2\mathrm{i})^2}\,\mathrm{d}z$, where C is the circle $|z-3\mathrm{i}| = 2$ (traversed in positive direction).
Hint: See Example 4 on page 897. Be careful about which singularities lie inside C.