MATH 3413 **Homework 4** **Due Thu, Feb 23, 2012**

Sec. 3.3: problems 14, 16, 23, 40, 43(a,b).

Sec. 3.5: problems 9, 21, 22, 26, 29, 32, 43(a).

Additional problem.

(a) Find the general solution of the equation

$$x'' + 6x' + 25x = f_0 \cos \omega t \tag{1}$$

describing a damped oscillator driven by a periodic external force of constant amplitude f_0 and angular frequency ω. (Find the values of the constants in $x_p(t)$, which are going to depend on ω.) Identify the *transient* part (the one that tends to 0 as $t \to \infty$) and the *persistent* (the one that survives as $t \to \infty$) part of the general solution.

(b) Show that, for any choice of constants α and β and of frequency ω, it is true that

$$\alpha \cos(\omega t) + \beta \sin(\omega t) = \gamma \cos(\omega t - \phi_0) \tag{2}$$

for some γ and ϕ_0. Write γ and ϕ_0 in terms of α and β, and then, conversely, write α and β as functions of γ and ϕ_0. This identity shows that each linear combination (with constant coefficients) of $\cos(\omega t)$ and $\sin(\omega t)$ is again a harmonic motion of frequency ω. The constants γ and ϕ_0 are often referred to as the *amplitude* of the motion and the *phase difference* between the external forcing and the motion of the system.

(c) Using (2), rewrite the persistent part $x_p(t)$ of the solution found in (a) in the form $x_p(t) = \gamma \cos(\omega t + \phi_0)$. Write the amplitude γ as a function of the external forcing frequency ω. assume that $f_0 = 1$. Find the value of ω for which the amplitude γ reaches its maximum value.

Hint: You have to show that the function $\gamma(\omega)$ giving the amplitude as a function of the external forcing frequency is

$$\gamma(\omega) = \frac{|f_0|}{\sqrt{(25 - \omega^2)^2 + (6\omega)^2}} \ .$$

You may use (without proving it) the fact that

$$\gamma'(\omega) = \frac{2\omega(7 - \omega^2)\,|f_0|}{[(25 - \omega^2)^2 + (6\omega)^2]^{3/2}} \ .$$