MATH 4433 **Homework 2** **Due 2/2/17 (Thursday)**

Ross, Section 3.1: Exercise 3.3 (on page 19).

Please write your proof succinctly and neatly, pointing out at each step which axiom (or a previous part of Theorem 3.1 of Ross, or some of the Mini-Theorems proved in class) you are using. In referring to the axioms, please use the notations (A1)–(A4), (M1)–(M4), or (DL) from the handout that I gave in Lecture 3 (this handout is also linked from Lecture 3 on the class web-site).

Additional Problem 1. Write down the negation of each of the following statements. *No explanation is necessary!*

(a) The function $f(x) = x^2 - 9$ is continuous at $x = 3$.

(b) x is in A or x is not in B.

(c) If $x < 7$, then $f(x)$ is not in C.

(d) If the sequence $\{a_n\}$ is convergent, then $\{a_n\}$ is monotone and bounded.

(e) Everyone likes Robert.

(f) $\exists\, x$ in B such that $f(x) > 7$.

(g) If $x > 5$, then $(f(x) < 3$ or $f(x) > 7)$.

Additional Problem 2. Determine the truth value of each statement (T or F), assuming that x and y are real numbers. If the statement is true, explain why; if the statement is false, give a counterexample. *Note that this is only an exercise in elementary logic and notations, do not refer to the axioms for real numbers discussed in class!*

(a) $\forall\, x$ and y, $\exists\, z$ such that $x + y = z$.

(b) $\forall\, x\ \exists\, y$ such that $\forall\, z$, $x + y = z$.

(c) $\exists\, x$ such that $\forall\, y\ \exists\, z$ such that $xz = y$.

(d) $\forall\, x$ and y, $\exists\, z$ such that $yz = x$.

Additional Problem 3. We say that a number is *irrational* if it is not rational. Some of the statements below are true, others are false. Prove of give a counterexample to each of the statements below. You may use that $\sqrt{2}$ is an irrational number.

(a) If x is irrational and y is irrational, then $x + y$ is irrational.

(b) If $x + y$ is irrational, then x is irrational or y is irrational.

Hint: This statement is true. To prove it, give a proof of its *contrapositive*, i.e., assume that the negation of the statement (x is irrational or y is irrational) is true, and show that this implies the negation of the statement ($x + y$ is irrational).

(c) If x is irrational and y is irrational, then xy is irrational.

(d) If xy is irrational, then x is irrational or y is irrational.

Hint: See the Hint to part (b).

Additional Problem 4. Consider the function $f : [1, \infty) \to [0, \infty)$ defined by $f(x) = x^3 - x$.

(a) Show that f is onto (i.e., surjective).

Hint: You may need to use the fact that $\lim_{x \to \infty} f(x) = \infty$ and the fact that f is continuous (so that the Intermediate Value Theorem holds).

(b) Show that f is 1-to-1 (i.e., injective).

Hint: Use tools from Calculus to show that f is strictly increasing.

Additional Problem 5. Suppose that $f : A \to B$ is a function, and let C be a subset of A.

(a) Give a counterexample to the following: $f(A \setminus C) \subseteq f(A) \setminus f(C)$.

Hint: The example $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ may be useful.

(b) Prove that $f(A) \setminus f(C) \subseteq f(A \setminus C)$.