MATH 4433 Homework 10a NOT Due 5/4/17 (Thursday)

Abbott, Section 6.2:

Exercises 6.2.1, 6.2.3, 6.2.6(b), 6.2.9, 6.2.11 (pages 180–182).

Remarks and hints:

- Exercise 6.2.9: In part (b), consider, e.g., $f_n : \mathbb{R} \to \mathbb{R} : x \mapsto x$, $g_n : \mathbb{R} \to \mathbb{R} : x \mapsto \frac{1}{n}$.
- Exercise 6.2.11: This is a more challenging problem (that is why it has a name).

  In part (a) you will conclude that g_n is continuous and $g_n \geq 0$ for all $n \in \mathbb{N}$, and that, for every $x \in K$, the sequence $(g_n(x))$ is decreasing with n, and $\lim_{n\to\infty} g_n(x) = 0$.

  To show that g_n converges uniformly to 0 on K, you need to show that for any $\varepsilon > 0$, one can find N such that $|g_n(x)| < \varepsilon$ for all $n \geq N$ and for all $x \in K$. In the statement of the problem the author has defined the sets $K_n = \{x \in K : g_n(x) \geq \varepsilon\}$. Each of these sets is compact (why?), and the sequence is nested: $K_1 \supseteq K_2 \supseteq K_3 \supseteq \cdots$. How is the uniform convergence of the sequence (g_n) to 0 related to whether the intersection $\bigcap_{n\in\mathbb{N}} K_n$ is empty or not?