MATH 2443 – Homework given on 9/26/09

Problem 1. In this problem you will use three methods to find the shortest distance from a point to a straight line in $\mathbb{R}^3$. 0 Let the position of the point be given by the vector $\mathbf{p} = \langle p_1, p_2, p_3 \rangle$, and the straight line be defined by the parametric equation $\mathbf{R}(t) = \mathbf{a} + \mathbf{b}t$, where $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$.

(a) Let $\Delta(t)$ be the square of the distance between the points $\mathbf{p}$ and $\mathbf{R}(t)$. This is a function of one variable, so you can find its minimum by applying the methods from Calculus I.

(b) Think geometrically (you may use dot or cross product of vectors, for example).

(c) Let $f(\mathbf{r}) = f(x, y, z)$ be the square of the distance from the arbitrary point $\mathbf{r} = \langle x, y, z \rangle \in \mathbb{R}^3$ and the given point $\mathbf{p}$. Minimize the function f subjected to the two constraints $g(x, y, z) = 0$ and $h(x, y, z) = 0$ such that the straight line $\mathbf{R}(t) = \mathbf{a} + \mathbf{b}t$ is the intersection of the surfaces

$$\mathcal{G} = \{(x, y, z) \in \mathbb{R}^3 : g(x, y, z) = 0\}$$

and

$$\mathcal{H} = \{(x, y, z) \in \mathbb{R}^3 : h(x, y, z) = 0\} \ .$$

Remark: You have to choose the functions g and h appropriately. You may assume that the vector $\mathbf{b}$ is in general position, i.e., that it is not collinear to any of the vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$. Make a simple choice (a complicated one will work as well, but the calculations may be long).