

MATH 2443 – Homework given on 9/26/09

Problem 1. In this problem you will use three methods to find the shortest distance from a point to a straight line in \mathbb{R}^3 . 0 Let the position of the point be given by the vector $\mathbf{p} = \langle p_1, p_2, p_3 \rangle$, and the straight line be defined by the parametric equation $\mathbf{R}(t) = \mathbf{a} + \mathbf{b}t$, where $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$.

- (a) Let $\Delta(t)$ be the square of the distance between the points \mathbf{p} and $\mathbf{R}(t)$. This is a function of one variable, so you can find its minimum by applying the methods from Calculus I.
- (b) Think geometrically (you may use dot or cross product of vectors, for example).
- (c) Let $f(\mathbf{r}) = f(x, y, z)$ be the square of the distance from the arbitrary point $\mathbf{r} = \langle x, y, z \rangle \in \mathbb{R}^3$ and the given point \mathbf{p} . Minimize the function f subjected to the two constraints $g(x, y, z) = 0$ and $h(x, y, z) = 0$ such that the straight line $\mathbf{R}(t) = \mathbf{a} + \mathbf{b}t$ is the intersection of the surfaces

$$\mathcal{G} = \{(x, y, z) \in \mathbb{R}^3 : g(x, y, z) = 0\}$$

and

$$\mathcal{H} = \{(x, y, z) \in \mathbb{R}^3 : h(x, y, z) = 0\} .$$

Remark: You have to choose the functions g and h appropriately. You may assume that the vector \mathbf{b} is in general position, i.e., that it is not collinear to any of the vectors \mathbf{i} , \mathbf{j} , and \mathbf{k} . Make a simple choice (a complicated one will work as well, but the calculations may be long).