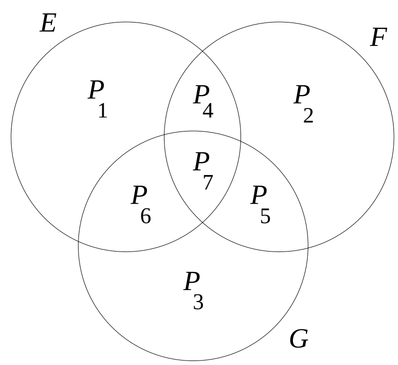
MATH 5453 Homework 1 Due Thu, Sep 4

Problems 9, 10, 12 from Section 1.3 of the book.

Remark: For Problem 12, you will need the concepts of symmetric difference of sets, $E \triangle F$, equivalence relation and equivalence classes (all defined in Section 0.1), and metric (defined in Section 0.6).

Hint to Problem 12(c): One very simple way of solving this problem is to represent the "distances" $\rho(E,G)$, $\rho(E,F)$, and $\rho(F,G)$ in terms of the measures of the disjoint sets P_j in the figure below. Note that P_j is the smallest "piece" in which "P_j" is written [i.e., $P_1 = E \cap F^c \cap G^c$, $P_4 = E \cap F \cap G^c$, …; in these notations, $E = P_1 \cup P_4 \cup P_6 \cup P_7$, etc.].



Additional problem 1. Let $(X, \mathcal{M}, \mu)$ be a measure space, and E_1, E_2, … be measurable sets. Prove *Boole's inequality*,

$$\mu\left(\bigcup_{j=1}^{\infty} E_j\right) \leq \sum_{j=1}^{\infty} \mu(E_j) \ .$$

Hint: Use induction and the result of Problem 9 of Section 1.3.

Additional problem 2. The notions of *limit superior* and *limit inferior* are defined and discussed in Section 0.1 of the book. Let the subsets E_n, F_n, and G_n of $\mathbb{R}$ (where $n \in \mathbb{N}$) be defined as

$$E_n := \left(-\infty, 5 + \frac{1}{n}\right) \ , \quad F_n := \left(-\infty, 5 - \frac{1}{n}\right) \ , \quad G_n := \left(-\infty, 5 + \frac{(-1)^n}{n}\right) \ .$$

Find $\limsup E_n$, $\liminf E_n$, $\limsup F_n$, $\liminf F_n$, $\limsup G_n$, $\liminf G_n$. Explain briefly your reasoning.