MATH 3113 – Homework assigned on 9/9/13

Problem 1. Consider the autonomous differential euqation

$$\frac{dx}{dt} = \mu x - x^3 \ , \tag{1}$$

where μ is a parameter. Let $f(x) := \mu x - x^3$ be the right-hand side of (1).

(a) If $\mu \leq 0$, show that the only equilibrium solution of the ODE (1) is $x(t) \equiv 0$, and it is stable. Sketch the graph of $f(x)$ and indicate how you came to your conclusion.

Hint: Computing $f'(x)$ will help you draw conclusions about the behavior of $f(x)$.

(b) If $\mu > 0$, show that the equilibrium $x_*(t) \equiv 0$ of the ODE (1) is now unstable, but there are two new equilibria, $x_{*1} = -\sqrt{\mu}$ and $x_{*2} = -\sqrt{\mu}$, which are stable. Again, sketch the graph of $f(x)$ for $\mu > 0$, and show on it how x changes with time.

(c) From your findings in parts (a) and (b), you can conclude that the qualitative nature of the solutions of the ODE (1) changes at $\mu = 0$ as the parameter μ increases, hence $\mu = 0$ is a bifurcation point for the ODE (1). In the (μ, x)-plane, plot the positions of the equilibrium solutions as functions of the parameter μ, for all values of μ. In your plot, denote the positions of the stable equilibria with a solid line, and the positions of the unstable equilibria with a dashed line.