

Hint to Problem 7.4/29

To find the Laplace transform of the product $tx'(t)$, use Theorem 2 on page 476 and the Corollary on page 454 to obtain the following:

$$\mathcal{L}\{tx'(t)\}(s) = -\mathcal{L}\{-tx'(t)\}(s) = -\frac{d}{ds}\mathcal{L}\{x'\}(s) = -\frac{d}{ds}[sX(s) - x(0)] ,$$

and similarly for $tx''(t)$. You will obtain that $X(s)$ satisfies the separable differential equation

$$(s + 1)X'(s) + 4X(s) = 0,$$

from which you have to find $x(t)$. Reading Example 5 on page 477 will be useful.

Please write your calculations in detail!