Axioms of ordered fields

(A0) **There exists an operation addition such that** $\forall x, y \in \mathbb{R},\ x + y \in \mathbb{R},$ **and:**

(A1) [**associativity of** $+$] $\forall x, y, z \in \mathbb{R},\ x + (y + z) = (x + y) + z$

(A2) [**commutativity of** $+$] $\forall x, y \in \mathbb{R},\ x + y = y + x$

(A3) [**existence of a unit element w.r.t.** $+$] $\exists\, 0 \in \mathbb{R}$ s.t. $x + 0 = x\ \ \forall x \in \mathbb{R}$

(A4) [**existence of an inverse element w.r.t.** $+$] $\forall x \in \mathbb{R}\ \exists\, (-x) \in \mathbb{R}$ s.t. $x + (-x) = 0$

(M0) **There exists an operation multiplication such that** $\forall x, y \in \mathbb{R},\ x \cdot y \in \mathbb{R},$ **and:**

(M1) [**associativity of** $\cdot$] $\forall x, y, z \in \mathbb{R},\ x \cdot (y \cdot z) = (x \cdot y) \cdot z$

(M2) [**commutativity of** $\cdot$] $\forall x, y \in \mathbb{R},\ x \cdot y = y \cdot x$

(M3) [**existence of a unit element w.r.t.** $\cdot$] $\exists\, 1 \in \mathbb{R}$ s.t. $1 \neq 0$ and $x \cdot 1 = x\ \ \forall x \in \mathbb{R}$

(M4) [**existence of an inverse element w.r.t.** $\cdot$] $\forall x \in \mathbb{R}$ with $x \neq 0\, \exists\, x^{-1} \in \mathbb{R}$ s.t. $x \cdot x^{-1} = 1$

(DL) [**distributive law**] $\forall x, y, z \in \mathbb{R},\ x \cdot (y + z) = x \cdot y + x \cdot z$

(O1) Given $x, y \in \mathbb{R}$, either $x \leq y$ or $y \leq x$

(O2) If $x \leq y$ and $y \leq x$, then $x = y$

(O3) [**transitivity**] If $x \leq y$ and $y \leq z$, then $x \leq z$

(O4) If $x \leq y$, then $x + z \leq y + z\ \ \forall z \in \mathbb{R}$

(O5) If $x \leq y$ and $0 \leq z$, then $x \cdot z \leq y \cdot z$