Calc 3 EXAM 3

FALL 2010

88 - 100	A	26.0 %
76 - 87	B	19.4 %
60 - 75	C	25.6 %
50 - 59	D	11.2 %
0 - 49	F	17.8 %

Median SCORE = 73

CALCULUS3 EXAM3

FALL 2010

NAME____________

REC.INSTR.____________

REC.TIME____________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK

(15) 1. A mass distribution occupies the region in the 1st octant that is enclosed by the surfaces $y = x$, $y = x^2$, $z = x$ and $z = 0$. The mass density function is $\delta = 120yz$ units of mass/unit volume. Calculate the total mass in the region.

$$\text{Mass} = \iiint_E 120yz\,dV \quad 3$$

$$= \int_0^1 \int_{x^2}^{x} \int_0^{x} 120yz\,dz\,dy\,dx \quad 3$$

$$= \int_0^1 \int_{x^2}^{x} 60yz^2 \Big|_{z=0}^{z=x} dy\,dx$$

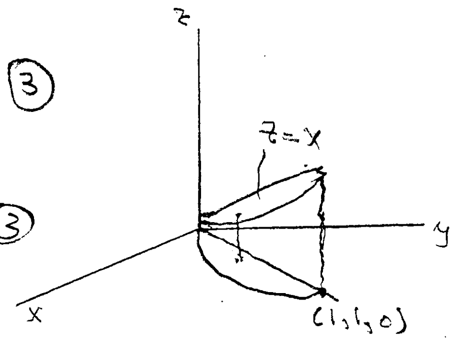
$$= \int_0^1 \int_{x^2}^{x} 60yx^2\,dy\,dx \quad 3$$

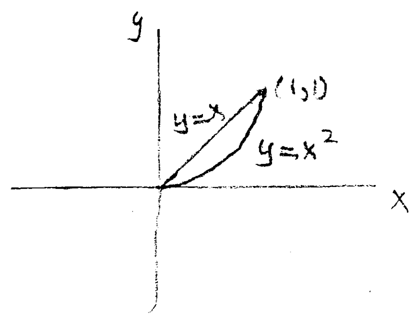
$$= \int_0^1 30y^2x^2 \Big|_{y=x^2}^{y=x} dx$$

$$= \int_0^1 (30x^4 - 30x^6)\,dx \quad 3$$

$$= 6x^5 - \frac{30x^7}{7} \Big|_0^1$$

$$= 6 - \frac{30}{7} = \frac{12}{7} \text{ units of mass} \quad 3$$





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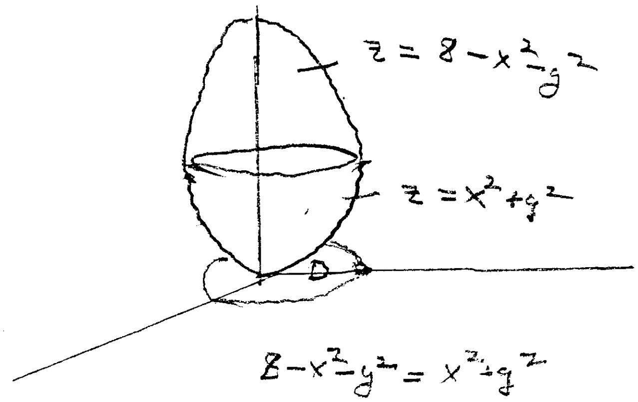
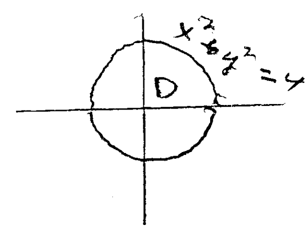
(15) 2. A mass distribution occupies the region enclosed by the surfaces $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$. The mass density function is $\delta = (x^2 + y^2)^{\frac{1}{2}}$ units of mass/unit volume. Calculate the total mass in the region.



$8 - x^2 - y^2 = x^2 + y^2$

gives $x^2 + y^2 = 4$



$$\text{Mass} = \iiint_E \sqrt{x^2+y^2}\, dV \quad \Big\} \text{ 3}$$

$$= \iint_D \int_{x^2+y^2}^{8-x^2-y^2} \sqrt{x^2+y^2}\, dz\, dA \quad \Big\} \text{ 3}$$

$$= \int_0^{2\pi}\int_0^2\int_{r^2}^{8-r^2} r\, dz \cdot r\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^2 r^2 z \Big|_{z=r^2}^{z=8-r^2} dr\, d\theta \qquad \text{5}$$

$$= \int_0^{2\pi}\int_0^2 (8r^2 \underbrace{- r^4 - r^4}_{-2r^4})\, dr\, d\theta$$

$$= \int_0^{2\pi} \left(\tfrac{8}{3}r^3 - \tfrac{2}{5}r^5\right)\Big|_{r=0}^{r=2} d\theta$$

$$= (2\pi)\left(\tfrac{64}{3} - \tfrac{64}{5}\right) \qquad \text{4}$$

$$= (128\pi)\left(\tfrac{1}{3} - \tfrac{1}{5}\right)$$

$$= \frac{256\pi}{15} \simeq 53.616$$

units of mass

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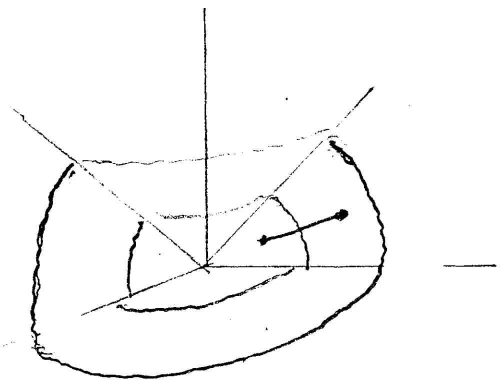
(15) 3. Find the volume of the region which is in the 1st octant, under the surface $z = (x^2 + y^2)^{\frac{1}{2}}$ and between the spheres of radius = 2 and radius = 1 with center the origin. <u>Use a triple integral in spherical coordinates</u>.



$$\text{Volume} = \iiint_E 1 \, dV \quad \Big\} \text{ 3}$$

$$= \int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_1^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta \quad \Big\} \text{ 6}$$

$$= \int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\rho^3}{3} \Big|_1^2 \sin\phi \, d\phi \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{7}{3} \sin\phi \, d\phi \, d\theta \quad \Big\} \text{ 3}$$

$$= \frac{7}{3} \int_0^{\frac{\pi}{2}} -\cos\phi \Big|_{\phi = \frac{\pi}{4}}^{\phi = \frac{\pi}{2}} d\theta$$

$$= \frac{7}{3} \cdot \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} d\theta$$

$$= \frac{7}{3} \cdot \frac{1}{\sqrt{2}} \cdot \frac{\pi}{2} = \frac{7\pi}{6\sqrt{2}} \simeq 2.59 \text{ units}^3 \quad \Big\} \text{ 3}$$

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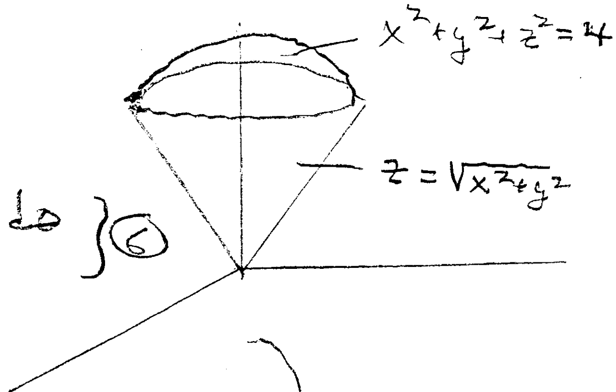
(15) 4. A mass distribution occupies the region which is above the surface $z = (x^2 + y^2)^{\frac{1}{2}}$ and inside $x^2 + y^2 + z^2 = 4$. The mass density function is $\delta = z^2$ units of mass/unit volume. Calculate the total mass in the region.



$$\text{Mass} = \iiint_E z^2\,dV \quad \Big\}\ 3$$

$$= \int_0^{2\pi}\int_0^{\pi/4}\int_0^{2} \underbrace{\rho^2\cos^2\phi \cdot \rho^2}_{\rho^4}\sin\phi\, d\rho\, d\phi\, d\theta \quad \Big\}\ 6$$

$$= \frac{32}{5}\int_0^{2\pi}\int_0^{\pi/4} \cos^2\phi\,\sin\phi\, d\phi\, d\theta$$

$$= \frac{32}{5}\int_0^{2\pi} -\frac{1}{3}\cos^3\phi \Big|_{\phi=0}^{\phi=\pi/4} = \frac{32}{15}\int_0^{2\pi}\left(-\frac{1}{2\sqrt{2}}+1\right) d\theta$$

$$= \frac{64\pi}{15}\left(1-\frac{1}{2\sqrt{2}}\right) \simeq 8.665 \text{ units of mass} \quad \Big\}\ 6$$

OR

$$\iint_D \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} z^2\,dz\,dA = \iint_D \int_r^{\sqrt{4-r^2}} z^2\,dz \cdot r\,dr\,d\theta$$

$$\left\{ \begin{array}{l} x^2+y^2+z^2=4 \\ z^2 = x^2+y^2 \\ \text{gives } x^2+y^2=2 \end{array} \right.$$

$$= \int_0^{2\pi}\int_0^{\sqrt{2}} \frac{1}{3}z^3 \Big|_{z=r}^{z=\sqrt{4-r^2}} r\,dr\,d\theta$$

$$= \frac{1}{3}\int_0^{2\pi}\int_0^{\sqrt{2}} \left[(4-r^2)^{3/2} r - r^4\right] dr\,d\theta$$

$$= \frac{1}{3}\int_0^{2\pi} \frac{2}{5}(4-r^2)^{\frac{5}{2}}\left(-\frac{1}{2}\right) - \frac{r^5}{5} \Big|_{r=0}^{r=\sqrt{2}} d\theta$$

$$= \frac{1}{15}\int_0^{2\pi} \left[-2^{\frac{5}{2}} - 2^{\frac{5}{2}} + 4^{\frac{5}{2}}\right] d\theta$$

$$= \frac{2\pi}{15}\left(4^{\frac{5}{2}} - 2\cdot 2^{5/2}\right) = \left(\frac{2\pi}{15}\right)\left(4^{\frac{5}{2}}\right)\left(1-\frac{1}{2\sqrt{2}}\right) = \frac{64\pi}{15}\left(1-\frac{1}{2\sqrt{2}}\right)$$

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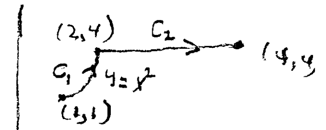
(25) 5. An object moves in the xy-plane from (1,1) to (4,4) by first going along $y = x^2$ from (1,1) to (2,4) and then going along $y = 4$ from (2,4) to (4,4).

a) If the force field $\mathbf{F} = y\mathbf{i} + 5xy\mathbf{j}$ acts on the object calculate the work done by $\mathbf{F}$.



$\text{Work} = \int_C y\,dx + 5xy\,dy$

C_1: $x = t,\ y = t^2;\quad 1 \le t \le 2,\quad dx = dt,\ dy = 2t\,dt$

$\int_{C_1} y\,dx + 5xy\,dy = \int_1^2 [t^2 + (5t)(t^2)(2t)]\,dt$

$= \int_1^2 [t^2 + 10t^4]\,dt = \frac{t^3}{3} + 2t^5 \Big|_1^2 = \frac{8}{3} + 64 - \frac{1}{3} - 2 = 62 + \frac{7}{3} = 64 + \frac{1}{3}$

7

$\frac{193}{3}$

C_2: $x = t,\ y = 4,\quad 2 \le t \le 4,\quad dx = dt,\ dy = 0$

6

$\int_{C_2} y\,dx + 5xy\,dy = \int_2^4 4\,dt = 4t\Big|_2^4 = 16 - 8 = 8$

So

Total Work $= 64 + \frac{1}{3} + 8 = 72 + \frac{1}{3}$ units of work

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b) If the force field $\mathbf{G} = (2x + y)\mathbf{i} + (x + 3y^2)\mathbf{j}$ also acts on the object calculate the work done by $\mathbf{G}$. Use the fundamental theorem of line integrals to evaluate ~~this line~~ integral.

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$\vec{G} = \nabla f$ with $f(x,y) = x^2 + xy + y^3$

So

$\text{Work} = f(4,4) - f(1,1) = 16 + 16 + 64 - 3 = 93$ units of work

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(15) 6. Use Green's theorem to evaluate the line integral

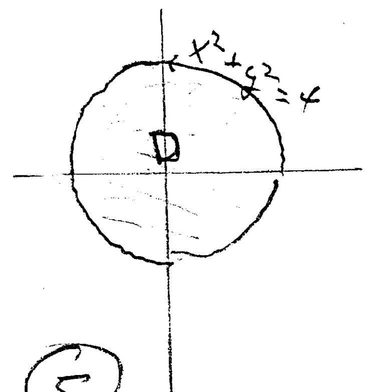
$$\int_C (2xy^2 + 2xy - y^2)dx + (x^2 + 2x + 2x^2y)dy$$

where C is the circle $x^2 + y^2 = 4$ directed counterclockwise.

$$\int_C \text{—} = \iint_D \left[\frac{\partial}{\partial x}(x^2+2x+2x^2y) - \frac{\partial}{\partial y}(2xy^2+2xy-y^2) \right] dA$$

$$= \iint_D \left[2x+2+4xy - (4xy+2x-2y) \right] dA$$

$$= \iint_D (2+2y)\, dA$$ 10



$$= \iint_D 2\, dA \quad \text{since } \iint_D y\, dA = 0$$

$$= 2\,\text{area } D = 8\pi$$ 5