Calculus 3 - EXAM I

FALL 2010

90-100	A ;	31.5 %
80-89	B ;	28.7 %
65-79	C ;	25.5 %
55-64	D ;	8.7 %
0-54	F ;	5.6 %

# of SCORES = 286

Median Score = 84

CALCULUS 3 EXAM 1
FALL 2010

NAME________________
REC.INSTR________________
REC. TIME________________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK

(25) 1. The points P(1,1,1), Q(2,0,3) and R(2,4,0) determine a triangle in 3-space.

a) find the angle at the vertex P in degrees

$\overrightarrow{PQ} = \langle 1,-1,2 \rangle, \ \overrightarrow{PR} = \langle 1,3,-1 \rangle \ \}$ ②

$\cos(\angle P) = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|} = \frac{1-3-2}{\sqrt{6}\sqrt{11}} = \frac{-4}{\sqrt{66}} \approx -.49236 \ \}$ ⑥

$\angle P \approx 119.496^\circ \ \}$ ②

b) find the area of the triangle

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 2 \\ 1 & 3 & -1 \end{vmatrix} = -5\vec{i} - (-3)\vec{j} + 4\vec{k} = \langle -5, 3, 4 \rangle \ \}$$ ④

$$\text{Area of triangle} = \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}| = \frac{1}{2}\sqrt{25+9+16} = \frac{1}{2}\sqrt{50} = \frac{5\sqrt{2}}{2} \text{ units}^2 \ \}$$ ④

c) find the equation of the plane which contains the triangle

$\overrightarrow{PQ} \times \overrightarrow{PR} = \langle -5, 3, 4 \rangle$ is a perp. vector for the plane } ②

Thus

$$(-5)(x-1) + 3(y-1) + 4(z-1) = 0$$

$$-5x + 3y + 4z = 2$$ ⑤

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(15) 2. a) find parametric equations for the line which passes through $(x,y,z) = (2,1,3)$ and is perpendicular to the plane

$$4x - 2y + z = 51$$

$\langle 4,-2,1\rangle$ is perp. to the plane and thus is parallel to the line } ⑥

so $\boxed{x = 2+4t,\ y = 1-2t,\ z = 3+t}$

b) find the point where the line you found in part a) intersects the plane

$$4x - 2y + z = 51$$

$4(2+4t) - 2(1-2t) + (3+t) = 51$ } ③

$8 + 16t - 2 + 4t + 3 + t = 51$

$21t + 9 = 51$

$21t = 42$

so $t = 2$ } ③

which gives

$(x,y,z) = (10,-3,5)$ } ③

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(30) 3. An object is moving in 3-space according to the parametric equations

$x = 2t$, $y = \sin t$, $z = t^2$ where t is the time.

Find, as functions of t,

④a) position vector **r** = $2t\vec{i} + \sin t\,\vec{j} + t^2\vec{k}$

④b) velocity vector **v** = $2\vec{i} + \cos t\,\vec{j} + 2t\,\vec{k}$

④c) acceleration vector **a** = $-\sin t\,\vec{j} + 2\vec{k}$

④d) speed = $|\vec{v}| = \sqrt{4+4t^2+\cos^2 t}$

⑤e) $a_T = \frac{d}{dt}(|\vec{v}|) = \dfrac{4t - \sin t\cos t}{\sqrt{4+4t^2+\cos^2 t}}$

f) curvature =

$$\vec{v}\times\vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & \cos t & 2t \\ 0 & -\sin t & 2 \end{vmatrix} = (2\cos t + 2t\sin t)\vec{i} - 4\vec{j} - 2\sin t\,\vec{k}$$

⑤ so

$$\left\{ \text{Curvature} = \frac{\left[16 + 4\sin^2 t + (2\cos t + 2t\sin t)^2\right]^{1/2}}{(4+4t^2+\cos^2 t)^{3/2}} \right.$$

④ g) $a_N = (\text{Curvature})|\vec{v}|^2$

$$= \left[\frac{16 + 4\sin^2 t + (2\cos t + 2t\sin t)^2}{4+4t^2+\cos^2 t}\right]^{1/2}$$

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(15) 4. An object is moving in 3-space in such a way that its acceleration vector as a function of the time t is $\mathbf{a} = 2t\mathbf{i} + e^{-t}\mathbf{k}$.
At time t = 0 its velocity vector is $\mathbf{v}(0) = \mathbf{i} + \mathbf{j}$
and its position vector is $\mathbf{r}(0) = \mathbf{j}$.

a) find the velocity vector as a function of t

$$\vec{v} = \int 2t\vec{i} + e^{-t}\vec{k}\,dt = t^2\vec{i} - e^{-t}\vec{k} + \vec{c} \quad \Big\}$$ ③

$t = 0$ gives $\vec{i} + \vec{j} = \vec{v}(0) = -\vec{k} + \vec{c}$ so $\vec{c} = \vec{i} + \vec{j} + \vec{k}$

and $\boxed{\vec{v} = (1 + t^2)\vec{i} + \vec{j} + (1 - e^{-t})\vec{k}}$ ③

b) find the position vector as a function of t

$$\vec{r} = \int \vec{v}\,dt = \left(t + \frac{t^3}{3}\right)\vec{i} + t\vec{j} + (t + e^{-t})\vec{k} + \vec{c} \quad \Big\}$$ ③

$t = 0$ gives $\vec{j} = \vec{r}(0) = \vec{k} + \vec{c}$ so $\vec{c} = \vec{j} - \vec{k}$

so $\boxed{\vec{r} = \left(t + \frac{t^3}{3}\right)\vec{i} + (1 + t)\vec{j} + (-1 + t + e^{-t})\vec{k}}$ ③

c) give the parametric equations for the motion

$$\left.\begin{aligned} x &= t + \frac{t^3}{3} \\ y &= 1 + t \\ z &= -1 + t + e^{-t} \end{aligned}\right\}$$ ③

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(15) 5. An object is moving in the xy-plane along the curve $y = \frac{1}{4}x^4$ from left to right. It is moving at a constant speed of 2 ft/sec.

$\frac{dy}{dx} = x^3$

$\frac{d^2y}{dx^2} = 3x^2$

a) find a_T and a_N when the object is at the point $(x, \frac{1}{4}x^4)$

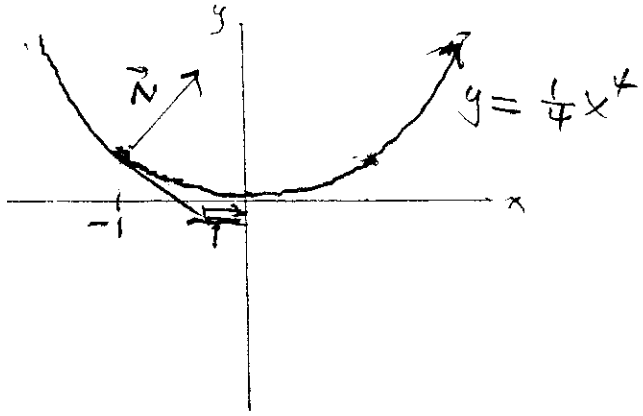
③ $\boxed{a_T = \frac{d}{dt}(\text{SPEED}) = 0}$ Since speed is constant

$a_N = (\text{Curvature})(\text{SPEED})^2 = 4\ \text{Curvature}$

$$\text{Curvature} = \frac{\left|\frac{d^2y}{dx^2}\right|}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}} = \frac{3x^2}{[1 + x^6]^{3/2}}$$

so $\boxed{a_N = \frac{12x^2}{[1 + x^6]^{3/2}}}$ ④

b) find the velocity vector and the acceleration vector when the object is at the point $(x,y) = (-1, \frac{1}{4})$



$\vec{v} = (\text{SPEED})\,\vec{T} = 2\vec{T}$

$$\text{Slope } \vec{T} = \left.\frac{dy}{dx}\right|_{x=-1} = \left.x^3\right|_{x=-1} = -1$$

so $\vec{T} = \frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}$

and $\boxed{\vec{v} = \sqrt{2}\,\vec{i} - \sqrt{2}\,\vec{j} \quad \text{at } (-1, \tfrac{1}{4})}$ ④

$$\vec{a} = \underset{0}{\cancel{a_T}}\vec{T} + a_N\vec{N} = \left.\frac{12x^2}{[1 + x^6]^{3/2}}\right|_{x=-1}\vec{N} = \frac{12}{2\sqrt{2}}\vec{N}$$

From $\vec{T}$ we see that
$\vec{N} = \frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$

so $\boxed{\vec{a} = \frac{6}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}\right) = 3\vec{i} + 3\vec{j} \quad \text{at } (-1, \tfrac{1}{4})}$ ④