Calculus 3 - EXAM 1

Spring 2010

90-100	A	;	47.8%
80-89	B	;	25.1%
70-79	C	;	15.3%
60-69	D	;	5.4%
0-59	F	;	6.4%

OF SCORES = 203

Median SCORE = 89

CALCULUS 3 SPRING 2010

EXAM 1

NAME

REC. INSTR

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK

(25) 1. The three points P(1,2,-1), Q(4,0,1) and R(-1,5,1) determine a triangle in 3-space.

a) find the angle of the triangle at the vertex P in degrees

10

$\vec{PQ} = \langle 3,-2,2 \rangle, \quad \vec{PR} = \langle -2,3,2 \rangle$

$\cos\theta = \dfrac{\vec{PQ}\cdot\vec{PR}}{|\vec{PQ}||\vec{PR}|} = \dfrac{-6-6+4}{\sqrt{9+4+4}\sqrt{4+9+4}} = -\dfrac{8}{17} \simeq -.47$

$\theta = \cos^{-1}\left(-\frac{8}{17}\right) = 118°$

4

b) find the area of the triangle

8

4

$\vec{PQ}\times\vec{PR} = \begin{vmatrix} i & j & k \\ 3 & -2 & 2 \\ -2 & 3 & 2 \end{vmatrix} = -10\vec{i} - 10\vec{j} + 5\vec{k}$

area of triangle $= \frac{1}{2}|\vec{PQ}\times\vec{PR}| = \frac{1}{2}\sqrt{100+100+25} = \frac{1}{2}\sqrt{225} = 7.5$ units^2

4

c) find the equation of the plane that contains the triangle

7

$\vec{PQ}\times\vec{PR} = \langle -10,-10,5 \rangle$ is perpendicular to the plane

3

so

$-10(x-1) - 10(y-2) + 5(z+1) = 0$

4

$-10x - 10y + 5z + 35 = 0$

$2x + 2y - z = 7$

NAME ____________________

(20) 2. Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

Find

10 a) the vector projection of **b** onto **a**, $\text{Proj}_{\mathbf{a}}\mathbf{b}$ =

$\vec{a} \cdot \vec{b} = 2 + 4 + 6 = 12$ 3

$|\vec{a}| = \sqrt{1+4+1} = \sqrt{6}$ 3

$\text{Proj}_{\vec{a}}\vec{b} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\vec{a} = \frac{12}{6}\vec{a} = 2\vec{a} = \langle 2,4,2\rangle$

4

5 b) $\mathbf{b} - \text{Proj}_{\mathbf{a}}\mathbf{b}$ = $\langle 2,2,6\rangle - \langle 2,4,2\rangle = \langle 0,-2,4\rangle$

5 c) the dot product

$(\mathbf{b} - \text{Proj}_{\mathbf{a}}\mathbf{b}) \cdot \mathbf{a}$ = $\langle 0,-2,4\rangle \cdot \langle 1,2,1\rangle = -4+4 = 0$

NAME_______________

(10) 3. a) Find parametric equations for the line through the point $(1,2,-3)$ which is perpendicular to the plane

$$2x - 3y + z = 10$$

$\vec{n} = \langle 2,-3,1\rangle$ is parallel to the line. (2

Thus

5

$$\begin{cases} x = 1+2t \\ y = 2-3t \\ z = -3+t \end{cases}$$ (3

b) Find the equation of the plane which contains the point $(2,-2,3)$ and is perpendicular to the line

$$(x-1)/4 = y/2 = (z-4)/(-1)$$

5

$\vec{n} = \langle 4,2,-1\rangle$ is perpendicular to the plane (2

Thus

$$4(x-2) + 2(y+2) - 1(z-3) = 0$$

$$4x + 2y - z = 1$$ (3

M

NAME____________________

(10) 4. Find the point of intersection of the two lines given by the parametric equations

L_1: $x = 4 + t,\ y = 2 - 2t,\ z = 4t$

L_2: $x = 2 + t,\ y = 2t,\ z = 1 - 2t$.

Note that to work this problem you should change the name of the parameter for one of the lines.

L_2: $x = 2 + s,\quad y = 2s,\quad z = 1 - 2s$

so $\underline{4 + t = 2 + s}$, $\quad 2 - 2t = 2s$, $\quad 4t = 1 - 2s$

$s = t + 2$ so $2 - 2t = 2t + 4$ so $-2 = 4t$

7 { Thus $t = -\frac{1}{2}$ and $s = \frac{3}{2}$ which gives $-2 = 1 - 3 = -2$ in 3rd eqn

Thus intersection point is

3
$$\begin{cases} x = 4 - \frac{1}{2} = \frac{7}{2} \\ y = 2 + 1 = 3 \\ z = (4)(-\frac{1}{2}) = -2 \end{cases}$$

NAME____________________

(25) 5. $x = t^2$, $y = t$, $z = t^2$ is a parametrized space curve .

Find as functions of t,

5 a) the position vector $\mathbf{r}(t)$ = $t^2\vec{i} + t\vec{j} + t^2\vec{k}$

5 b) tangent vector $\mathbf{r}'(t)$ = $2t\vec{i} + \vec{j} + 2t\vec{k}$

7 c) unit tangent vector $\mathbf{T}(t)$ = $\dfrac{\vec{r}'(t)}{|\vec{r}'(t)|}$, $|\vec{r}'(t)| = \sqrt{1+8t^2}$

$$\vec{T} = \frac{2t}{\sqrt{1+8t^2}}\vec{i} + \frac{1}{\sqrt{1+8t^2}}\vec{j} + \frac{2t}{\sqrt{1+8t^2}}\vec{k}$$

8 d) curvature =

$$\vec{r}''(t) = 2\vec{i} + 2\vec{k}$$

$$\vec{r}' \times \vec{r}'' = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2t & 1 & 2t \\ 2 & 0 & 2 \end{vmatrix} = 2\vec{i} - 2\vec{k}$$

$$\text{Curvature} = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|^3} = \frac{\sqrt{8}}{(1+8t^2)^{3/2}}$$

(10) 6. Find $\mathbf{r}(t)$ if

$\mathbf{r}'(t)$ = $2\cos t\ \mathbf{i} + 2t\ \mathbf{j} + 2\sin t\ \mathbf{k}$ and $\mathbf{r}(0) = \mathbf{j}$.

6 $\{\ \vec{r}(t) = 2\sin t\ \vec{i} + t^2\vec{j} - 2\cos t\ \vec{k} + \vec{C}$

$\vec{j} = \vec{r}(0) = -2\vec{k} + \vec{C}$ so $\vec{C} = \vec{j} + 2\vec{k}$

4 and

$\vec{r}(t) = 2\sin t\ \vec{i} + (1+t^2)\vec{j} + (2-2\cos t)\vec{k}$