Q1][15 points] Evaluate the following triple integral by first sketching the region of integration, and then converting it to a spherical coordinates integral.

\[
\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{\sqrt{2-x^2-y^2}}^{z} dz \, dx \, dy
\]

Q2][20 points] Write down the equation in the statement of Green’s theorem, indicating what the various parts of it stand for.
Compute \( \int_C \mathbf{F} \cdot d\mathbf{r} \) directly, where \( \mathbf{F} = (-x^2y^2, xy) \) and \( C \) is the positively oriented boundary of the region bounded by the \( y \)-axis, the line \( y = 1 \), and the curve \( y = \sqrt{x} \).
Use Green’s theorem to compute the path integral above by a second method. Compare your answers.

Q3][20 points] State the fundamental theorem for path integrals.
Let \( \mathbf{F} = (ye^{yz} \cos(xy), ze^{yz} \sin(xy) + xe^{yz} \cos(xy), ye^{yz} \sin(xy)) \). Show that \( \text{curl}(\mathbf{F}) = 0 \).
Find a function \( f \) so that \( \mathbf{F} = \nabla f \).
Use the fundamental theorem to give a quick computation of the path integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where \( C \) is the straight line curve from \((0, e, \pi/2)\) to \((\pi, 1/2, 0)\).

Q4][5 points] Determine (giving reasons) whether the following vector field has positive, negative or zero divergence.