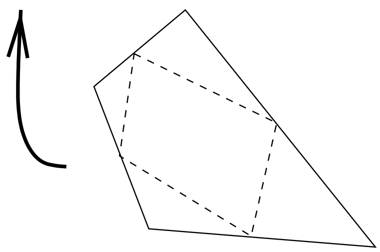
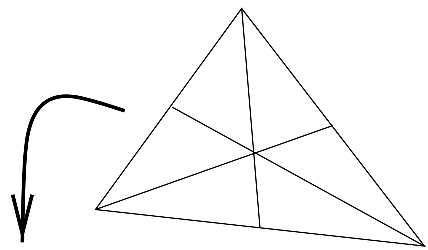
Math 2433-001   Calculus III
Applications of Vectors

Q1]... Given any quadrilateral $ABCD$, prove that one always obtains a parallelogram by drawing line segments between the midpoints of successive edges.





Q2]... A *median* of a triangle is a line connecting a vertex of the triangle to the midpoint of the opposite edge. Prove the at the three medians of a triangle intersect in a common point (which divides each median into a 1/3: 2/3 ratio).

Q3]... You and your partner are on an expedition to uncover an ancient buried treasure. One night your partner and you finally decipher the old map of the burial site and the instructions. There are n (you forget the exact number now) palm trees on the map, numbered from 1 through n. The instructions decipher to read as follows:

To find the treasure, start at tree 1 and walk half way towards tree 2, now turn and walk 1/3 way towards tree 3, then turn and walk 1/4 way towards tree 4, etc.

The following morning your partner and the map are both gone! You use your impeccable Calculus I and II skills to compute the optimal route to the burial site (minimizing total time taken) and arrive there a full hour ahead of your evil partner. You locate the n trees, but to your horror, you find that you cant remember which tree was labeled the first, which the second etc! Suddenly, you realize that a simple application of vectors from Calculus III ensures that **it doesn't matter which tree is labeled 1, which is labeled 2, etc**. You quickly uncover the treasure, and rush home to share half with your old professor.

Prove the boldface statement above! How does it generalize Q2] above?