Calculus III [2433–001] Extra Questions I

Q1]... We've seen in Q32, pge 624, that $1 + 1/2 + 1/3 + \cdots + 1/n - \ln n$ converges to Euler's constant γ. We can write this as

$$h_n \;=\; 1 + 1/2 + 1/3 + \cdots + 1/n \;=\; \ln n + \gamma + \epsilon_n$$

where the error term ϵ_n tends to 0 as $n \to \infty$. We denote the sum by h_n since it represents the nth partial sum of the *harmonic series*.

We can use this to compute the sum of the series

$$\sum_{n=1}^{\infty} (-1)^{n-1} 1/n \;=\; 1 - 1/2 + 1/3 - 1/4 + \cdots$$

as follows. **Copy down this argument and fill in the details**.

Step 1: The $2n$th partial sum is just

$$s_{2n} \;=\; 1 - 1/2 + \cdots + 1/(2n-1) - 1/(2n)\,.$$

This is just **(why?)**

$$[1 + \cdots + 1/(2n)] - 2[1/2 + 1/4 + \cdots + 1/(2n)]\,.$$

Step 2: We now use the h_n estimates above to get **(why?)**

$$s_{2n} \;=\; h_{2n} - h_n \;=\; [\ln(2n) + \gamma + \epsilon_{2n}] - [\ln(n) + \gamma + \epsilon_n]\,.$$

Step 3: Finally we can take $\lim_{n\to\infty} s_{2n}$. Note that the γ's cancel and the ϵ's tend to 0, and so we get a sum of $\ln 2$ (**why?**) as $n \to \infty$. That is

$$\sum_{n=1}^{\infty} (-1)^{n-1} 1/n \;=\; \ln(2)\,.$$

Q2]... Arguing as above, do Q27 on page 682.

Q3–6]... Q20, pge 681; Q25, Q28, pge 682; Q32 pge 683.

Hint for Q20: We know that the geometric series $\sum \frac{1}{2^n}$ and $\sum \frac{1}{3^n}$ give us series whose terms are reciprocals of powers of two and three respectively. How do we get **products** of powers of two and three?