Calculus III [2433–001] Extra Questions I

Q1]... We’ve seen in Q32, pge 624, that $1 + 1/2 + 1/3 + \cdots + 1/n - \ln n$ converges to Euler’s constant $\gamma$. We can write this as

$$h_n = 1 + 1/2 + 1/3 + \cdots + 1/n = \ln n + \gamma + \epsilon_n$$

where the error term $\epsilon_n$ tends to 0 as $n \to \infty$. We denote the sum by $h_n$ since it represents the $n$th partial sum of the harmonic series.

We can use this to compute the sum of the series

$$\sum_{n=1}^{\infty} (-1)^{n-1} 1/n = 1 - 1/2 + 1/3 - 1/4 + \cdots$$

as follows. Copy down this argument and fill in the details.

**Step 1:** The $2n$th partial sum is just

$$s_{2n} = 1 - 1/2 + \cdots + 1/(2n-1) - 1/(2n)$$

This is just (why?)

$$[1 + \cdots + 1/(2n)] - 2[1/2 + 1/4 + \cdots + 1/(2n)].$$

**Step 2:** We now use the $h_n$ estimates above to get (why?)

$$s_{2n} = h_{2n} - h_n = [\ln(2n) + \gamma + \epsilon_{2n}] - [\ln(n) + \gamma + \epsilon_n].$$

**Step 3:** Finally we can take $\lim_{n \to \infty} s_{2n}$. Note that the $\gamma$’s cancel and the $\epsilon$’s tend to 0, and so we get a sum of $\ln 2$ (why?) as $n \to \infty$. That is

$$\sum_{n=1}^{\infty} (-1)^{n-1} 1/n = \ln(2).$$

Q2]... Arguing as above, do Q27 on page 682.

Q3–6]... Q20, pge 681; Q25, Q28, pge 682; Q32 pge 683.

**Hint for Q20:** We know that the geometric series $\sum 1/2^n$ and $\sum 1/3^n$ give us series whose terms are reciprocals of powers of two and three respectively. How do we get **products** of powers of two and three?