Pge 347 Q7 Hwk 15

$$\vec{X}' = \begin{pmatrix} 2 & 0 & 0 \\ -7 & 9 & 7 \\ 0 & 0 & 2 \end{pmatrix} \vec{X}$$

$$\det \begin{pmatrix} 2-\lambda & 0 & 0 \\ -7 & 9-\lambda & 7 \\ 0 & 0 & 2-\lambda \end{pmatrix} = 0 \Rightarrow (2-\lambda)\det\begin{pmatrix} 9-\lambda & 7 \\ 0 & 2-\lambda \end{pmatrix} = 0$$

$$\Rightarrow (2-\lambda)(9-\lambda)(2-\lambda) = 0$$

$\lambda = 2, 2, 9$

$\boxed{\lambda = 2}$

$$\begin{pmatrix} 0 & 0 & 0 \\ -7 & 7 & 7 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-7x + 7y + 7z = 0$

$\Rightarrow x = y + z$ ← a plane

$\Rightarrow$ 2 independent 2-eigenvectors

e.g. let $y = 0$ & $z = 1$ $\Rightarrow x = 0 + 1 = 1$

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

Let $y = 1$ & $z = 0$ $\Rightarrow x = 1 + 0 = 1$

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$\boxed{\lambda = 9}$

$$\begin{bmatrix} -7 & 0 & 0 \\ -7 & 0 & 7 \\ 0 & 0 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-7x = 0$ $x = 0$

$-7z = 0$ $z = 0$ $\Rightarrow \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$\vec{X}(t) = c_1 e^{2t}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + c_2 e^{2t}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_3 e^{9t}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

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$$\det\begin{pmatrix}-1-\lambda & 0 & 1\\ 0 & -1-\lambda & 1\\ 1 & -1 & -1-\lambda\end{pmatrix} = 0 \Rightarrow (-1-\lambda)\det\begin{pmatrix}-1-\lambda & 1\\ -1 & -1-\lambda\end{pmatrix} + 1\cdot\det\begin{pmatrix}0 & -1-\lambda\\ 1 & -1\end{pmatrix} = 0$$

$$\Rightarrow (-1-\lambda)\left((-1-\lambda)^2 + 1\right) + 1(1+\lambda) = 0$$

$$\Rightarrow (-1-\lambda)\left((-1-\lambda)^2 \cancel{+1}\cancel{-1}\right) = 0 \Rightarrow (1+\lambda)^3 = 0 \qquad \lambda = -1, -1, -1$$

$$\begin{pmatrix}0 & 0 & 1\\ 0 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}0\\ 0\\ 0\end{pmatrix} \qquad \begin{matrix}z=0\\ x-y=0\end{matrix} \quad (x=y) \rightsquigarrow \begin{pmatrix}1\\ 1\\ 0\end{pmatrix} = \vec{V}_1$$

Only one eigenvector direction. Need 2 generalized eigenvectors.

$$\begin{pmatrix}0 & 0 & 1\\ 0 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}1\\ 1\\ 0\end{pmatrix} \Rightarrow \begin{cases} z=1 \\ x-y=0 \Rightarrow x=y\end{cases} \qquad \begin{pmatrix}1\\ 1\\ 1\end{pmatrix} = \vec{V}_2$$

$$\begin{pmatrix}0 & 0 & 1\\ 0 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}1\\ 1\\ 1\end{pmatrix} \Rightarrow \begin{matrix}z=1\\ x-y=1\end{matrix} \qquad \begin{matrix}x=1+y\\ z=1\end{matrix} \qquad \begin{pmatrix}1\\ 0\\ 1\end{pmatrix} = \vec{V}_3$$

$$\vec{V}_3 \xrightarrow{(A-\lambda I)} \vec{V}_2 \xrightarrow{(A-\lambda I)} \vec{V}_1 \xrightarrow{(A-\lambda I)} \vec{0}$$

$$\vec{X}(t) = c_1 e^{-t}\begin{pmatrix}1\\ 1\\ 0\end{pmatrix} + c_2 e^{-t}\left(\begin{pmatrix}1\\ 1\\ 1\end{pmatrix} + t\begin{pmatrix}1\\ 1\\ 0\end{pmatrix}\right) + c_3 e^{-t}\left(\begin{pmatrix}1\\ 0\\ 1\end{pmatrix} + t\begin{pmatrix}1\\ 1\\ 1\end{pmatrix} + \frac{t^2}{2}\begin{pmatrix}1\\ 1\\ 0\end{pmatrix}\right)$$