

1. Trig Addition, Half Angle.

$$\begin{aligned}\cos(A \pm B) &= \cos(A)\cos(B) \mp \sin(A)\sin(B) \\ \cos(2A) &= 2\cos^2(A) - 1 \\ \cos^2(x) &= (1 + \cos(2x))/2 \\ \sin(A \pm B) &= \sin(A)\cos(B) \pm \cos(A)\sin(B)\end{aligned}$$

$$\begin{aligned}\cos(2A) &= \cos^2(A) - \sin^2(A) \\ \sin^2(x) &= (1 - \cos(2x))/2 \\ \sin(2x) &= 2\sin(x)\cos(x)\end{aligned}$$

2. Hyperbolic.

$$\sinh(x) = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh(x) = \frac{1}{2}(e^x + e^{-x})$$

3. Integration by Parts.

$$\int u dv = uv - \int v du$$

4. Integration by substitution.

$$\int f(u(x)) \frac{du}{dx} dx = \int f(u) du$$

5. Inverse Trig.

$$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$$

6. Trig Substitutions.

For  $\sqrt{a^2 - x^2}$  use  $x = a \sin(\theta)$

For  $\sqrt{a^2 + x^2}$  use  $x = a \tan(\theta)$

For  $\sqrt{x^2 - a^2}$  use  $x = a \sec(\theta)$

7. Some integrals.

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int \tan(x) dx = \ln|\sec(x)| + C$$

$$\int \sec(x) dx = \ln|\sec(x) + \tan(x)| + C$$

8. First order linear ODE  $y' + p(x)y = q(x)$  can be solved by first multiplying across by an integrating factor

$$I = e^{\int p dx}$$

9. The equation  $M(x, y)dx + N(x, y)dy = 0$  is said to be exact if  $M_y = N_x$ . If it is exact, it can be solved by antidifferentiating  $M$  with respect to  $x$  and  $N$  with respect to  $y$  to obtain  $F(x, y)$  and then setting  $F(x, y) = C$ .

10. An ODE of the form  $y' = f(ax + by + c)$  can be solved by first making a substitution  $v = ax + by + c$ .

11. An ODE of the form  $y' = f(y/x)$  can be solved by first making a substitution  $v = y/x$ .

12. The Bernoulli equation  $y' + p(x)y = q(x)y^n$  can be solved by first making a substitution  $v = \frac{1}{y^{n-1}}$ .