

## Topics for final exam

1. Separable equations, first order linear equations and related equations. Exact equations
2. Linear equations, homogeneous and non-homogeneous equations.
3. Solution of constant coefficient homogeneous linear equations by the characteristic equation method.
4. Independent solutions and the general solution to an  $n$ th order homogeneous linear ODE.
5. Solution of certain types of non-homogeneous linear ODEs by the method of undetermined coefficients and by the method of variation of parameters.
6. Applications to mixing problems, population modeling, mechanical vibrations and electrical circuits.
7. Understanding of underdamped, critically damped and overdamped motion. Forced motion and resonance.
8. A knowledge of matrices, matrix multiplication and addition, column vectors, determinants of square matrices, the identity matrix.
9. Linear systems of ODEs, and the ability to encode a linear system using the language of matrices and column vectors.
10. Linear systems arising from other areas of mathematics or physics/engineering; “real-world” examples.
11. Non-homogeneous and homogeneous linear systems.
12. Superposition principle: General solution of a non-homogeneous linear system is the sum of the general solution to the associated homogeneous system and a particular solution of the non-homogeneous system.
13. Solving simple linear systems by elimination (operator determinant method).
14. Solving homogeneous linear systems by the eigenvalue–eigenvector method.
  - Compute eigenvectors of a square matrix via the characteristic equation.
  - Find eigenvectors  $\mathbf{v}_i$  corresponding to eigenvalues  $\lambda_i$ .
  - $\mathbf{x}_i = e^{\lambda_i t} \mathbf{v}_i$  are independent solutions.
15. Working with complex eigenvalues and eigenvectors.
16. Working with generalized eigenvectors.
17. A phase plane portrait gallery for systems with  $2 \times 2$  matrices.
18. Mechanical vibration examples, and systems of the form  $\frac{d^2 \mathbf{X}}{dt^2} = A \mathbf{X}$ .
19. The matrix  $\Phi(t)$  of fundamental solutions to a homogeneous linear system and its relationship with the matrix exponential  $e^{At}$ .
20. Alternative methods of computing matrix exponentials for special matrices.
21. Using the variation of parameters method to solve non-homogenous linear systems.
22. Definition of Laplace transform(LT), and using basic properties of LT to compute transforms of various functions,
23. Know why the various properties of LT are true.
24. Use LT to solve an initial value problem (IVP). This may involve algebra techniques such as partial fractions.