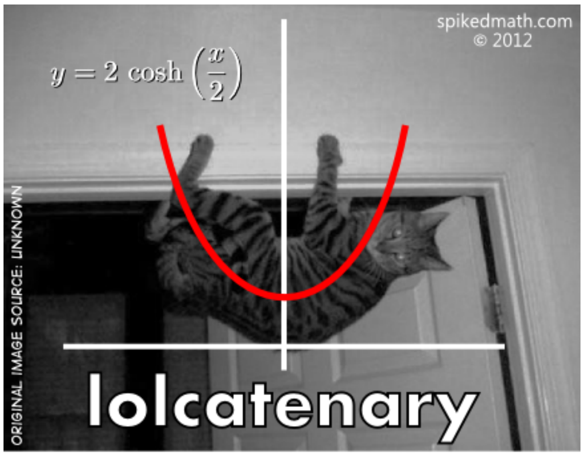
Friday 02/12/2016 Midterm I 50 minutes

Name: Student ID:

Instructions.

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. **Show all the steps of your work clearly. A final answer on its own is not enough for full points.**

Question	Points	Your Score
Q1	20	
Q2	20	
Q3	20	
Q4	20	
Q5	20	
TOTAL	100	



In physics and geometry, the lolcatenary is the curve that an idealized hanging lolcat assumes under its own weight when supported only at its ends.

Miscellaneous expressions and definitions.

1. **Trig Addition, Half Angle.**

   $\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$ $\cos(2A) = \cos^2(A) - \sin^2(A)$
   $\cos(2A) = 2\cos^2(A) - 1$ $\cos(2A) = 1 - 2\sin^2(A)$ $\sin^2(x) = (1 - \cos(2x))/2$
   $\cos^2(x) = (1 + \cos(2x))/2$ $\sin(2x) = 2\sin(x)\cos(x)$
   $\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$

2. **Hyperbolic.**

   $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$ $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$

3. **Integration by Parts.**

   $\int u\,dv = uv - \int v\,du$

4. **Integration by substitution.**

$$\int f(u(x))\frac{du}{dx}\,dx = \int f(u)\,du$$

5. **Inverse Trig.**

   $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$ $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}(\frac{x}{a})$

6. **Trig Substitutions.**

   For $\sqrt{a^2 - x^2}$ use $x = a\sin(\theta)$
   For $\sqrt{a^2 + x^2}$ use $x = a\tan(\theta)$
   For $\sqrt{x^2 - a^2}$ use $x = a\sec(\theta)$

7. **Some integrals.**

$$\int \frac{dx}{x} = \ln|x| + C \qquad \int \tan(x)\,dx = \ln|\sec(x)| + C$$

$$\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$$

8. **First order linear ODE** $y' + p(x)y = q(x)$ can be solved by first multiplying across by an integrating factor

$$I = e^{\int p\,dx}$$

9. The equation $M(x,y)dx + N(x,y)dy = 0$ is said to be **exact** if $M_y = N_x$. If it is exact, it can be solved by antidifferentiating M with respect to x and N with respect to y to obtain $F(x,y)$ and then setting $F(x,y) = C$.

10. An ODE of the form $y' = f(ax + by + c)$ can be solved by first making a substitution $v = ax + by + c$.

11. An ODE of the form $y' = f(y/x)$ can be solved by first making a substitution $v = y/x$.

12. The **Bernoulli equation** $y' + p(x)y = q(x)y^n$ can be solved by first making a substitution $v = \frac{1}{y^{n-1}}$.

13. Some **second order ODEs** can be solved by making the substitution $v = y'$.

Q1]. . . [20 points] Find the general solution to the ODE.

$$\frac{dy}{dx} = ye^x$$

Find the particular solution to the IVP.

$$\frac{dy}{dx} = ye^x \qquad\qquad y(0) = 3e$$

Q2]. . . [20 points] Find the general solution to the ODE.

$$x\frac{dy}{dx} + 2y \;=\; x^2 + 1$$

Q3]. . . [20 points] Find the general solution to the following ODE.

$$\frac{dy}{dx} = (x + y + 4)^2$$

Q4]. . . [20 points] Determine if the following ODE is exact.

$$(6x - y + 3)dx + (4y - x)dy \;=\; 0$$

Find the general solution to the ODE above.

Q5]... [20 points] Newton's Law of motion and Hooke's Law of the spring combine to give the following equation describing the motion of a mass m under a spring:

$$m\frac{d^2x}{dt^2} = -kx$$

Here t represents time, the motion is along the x–axis, the spring is in equilibrium (neither stretched nor compressed) at $x = 0$, and k is a positive constant that depends on the particular spring.

By making an appropriate substitution, derive the *law of conservation of energy* for this motion.