MATH 2513-002 Midterm I topics sheet

1. Know what the sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$ are.
2. Know the definitions of *odd* and *even* integers.
3. Can prove statements about even/odd-ness of powers, products, sums of odd/even integers.
4. Can prove that various numbers including $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt[3]{2}$, $\sqrt[3]{3}$ are all irrational.
5. Know the logical expressions $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$. Know their english equivalents, their truth tables, and be able to compute the truth table of compound expressions.
6. Know about predicates and quantifiers; in particular $(\forall x \in U)P(x)$ and $(\exists x \in U)P(x)$. Know how to interpret statements with nested quantifiers, and how to understand negations of quantified statements.
7. Be aware of different types of proofs: direct proofs (from hypothesis to conclusion), proofs via a logical equivalence (e.g., proof of the contrapositive statement), proofs using cases, and proofs by contradiction.
8. Understand the difference between converse and the contrapositive of a conditional statement.
9. Be able to prove simple properties of the real numbers as consequences of the list of properties in Table 1.2.
   - 0 is the only additive identity
   - 1 is the only multiplicative identity
   - additive inverses are unique
   - multiplicative inverses (of non-zero numbers) are unique
   - $0a = 0$ for all $a \in \mathbb{R}$
   - $a(-b)$ and $(-a)b$ are both additive inverses of ab for all $a, b \in \mathbb{R}$.
   - $(-a)(-b) = ab$ for all $a, b \in \mathbb{R}$.
   - If $a \neq 0$ and $b \neq 0$ are real numbers, then $ab \neq 0$.
10. Know the definition of $a \mid b$.
11. Know basic properties (and their proofs) of $a \mid b$.
    - If $a \mid b$ and $a \mid c$, then $a \mid (xb + yc)$ for all $x, y \in \mathbb{Z}$.
    - $m \mid a$ if and only if $a \equiv 0 \mod m$.
12. Know the definition of $a \equiv b \mod m$.
13. Know basic properties (and their proofs) of $a \equiv b \mod m$.
    - $a \equiv a \mod m$.
    - $a \equiv b \mod m$ implies $b \equiv a \mod m$.
    - $a \equiv b \mod m$ and $b \equiv c \mod m$ implies $a \equiv c \mod m$.
    - $a \equiv x \mod m$ and $b \equiv y \mod m$ implies $a + b \equiv x + y \mod m$.
    - $a \equiv x \mod m$ and $b \equiv y \mod m$ implies $ab \equiv xy \mod m$.
14. Know some early applications of $\equiv \mod m$, such as tests for divisibility.